ELEMENTS

OF

PLANE ASTRONOMY,

BY

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PREFACE.

THIS work is altogether designed for the use of students in the Dublin University, but as it may fall into the hands of some not aware of this circumstance, and who may expect many things not found therein, and may meet with other things to them apparently unsuitable, or insufficiently illustrated, it is necessary to give some explanation.

A treatise on astronomy professing to be complete, ought, in the first place, to abound with examples. In a treatise, however, merely designed to teach the outlines of the science, and to point out what may incite and lead to further inquiry, such examples are unnecessary. In the necessarily limited portion of time devoted to astronomical science in the course of a University education, a multitude of examples would, to the mass of students, be perfectly useless. It would be, therefore, improper to increase thereby the size of this volume, which has been prepared at the request of the college for their use.

Again, it may be said, that more matter was introduced than was absolutely necessary; that it was unnecessary, for instance, to introduce the subject of astro-

nomical instruments. It may be answered in reply to this, that their introduction is not only a benefit to the student, but an assistance to the Astronomical Professor. The latter, in his annual lectures, has an opportunity of explaining and illustrating the uses of the different instruments used in the practice of astronomy, to which lectures the students have free access; and the short preparatory account of the instruments given in this volume enables him to give, with much greater effect, a more minute explanation.

It may not be irrelevant to mention, that the greater part of the substance of this volume, according to its present arrangement, has been given by the author in his annual lectures since the year 1799, as Professor, and that the first sixteen chapters have been actually in the hands of the students since [illegible], having been then for the first time *printed* for their use.

The student who is anxious for a more extended knowledge of plane astronomy, and is desirous of familiarising himself with astronomical computations, will be at no loss for assistance. The works of Professors Vince and Woodhouse will afford him very extensive information. The different publications, too, under the sanction of the Board of Longitude, more especially the Nautical Almanack, will form a most useful practical aid to the student who desires it. In addition to these and other valuable British publications, the student may avail himself of a multitude of foreign works on the subject of astronomical science. The transactions of learned

cieties, both at home and abroad, constitute a third very extensive source of information on this subject.

Works on trigonometry are so numerous, and its applications so readily referred to, that in common instances it has not been thought necessary to particularize any author. In the Appendix, for which a more extended knowledge of trigonometry is necessary, the treatises of Professor Woodhouse and Mr. Luby have been quoted. These works will be found quite sufficient for obtaining the preparatory knowledge of trigonometry necessary for the more difficult parts of astronomy.

*** The author had intended to prepare some additions for this work, particularly on the subject of double stars and comets, and on the temperature of the earth and its variations. His distressing and tedious illness, with the harassing duties of his ministry, unfortunately prevented him from fulfilling this intention.

It is now too late, with regard to the present edition, to think of supplying the deficiency. The knowledge of scientific men on the subjects alluded to is moreover still in a very imperfect state, and the discussion of them may therefore be fairly reserved for a future edition. For a highly interesting detail of all that has been done in these matters, as well as of the nature of the questions themselves, and the various modes of subjecting them to the tests of observation and calculation, the reader is referred to the astronomical treatise in Lardner's Cyclopedia, by Sir J. Herschel.

ADDENDA ET CORRIGENDA.

Page xvii line 18, insert commas after the words "moon" and "planets."
—— 4, —— 12, *for* in *read* on.
—— 25, —— 17 The interposition of large opaque bodies revolving about them is assigned by Laplace as another explanation of this phenomenon
—— 43, —— 23, *for* precison *read* precision
—— 60, —— 17, *for* 56″ 2 *read* 50″ 2.
—— 77, —— 17 This law of Professor Bode would also fix a planet at the earth's distance from the sun, and thus it affords an argument for the earth's annular motion Similar laws have been shewn by Mr. Challis of Cambridge, to exist for the satellites, thus for Jupiter's satellites, for example:

$$\begin{array}{llll} \text{1st.} & \ldots & 7 & = 7 \\ \text{2nd} & \ldots & 7+4 & = 11 \\ \text{3rd} & . & 7+4\left(\frac{5}{2}\right) & = 17 \\ \text{4th.} & \ldots\ldots & 7+4\left(\frac{5}{2}\right)^2 & = 32 \end{array}$$

See Cambridge Phil. Trans. vol. iii.

—— 102, —— 17, *for* 1834, *read* 1835. By the calculations of Pontecoulant and Damoiseau, the return of this comet to its perihelion was fixed, by the former to the 7th, and by the latter to the 4th of November, 1835
—— 109, —— 15, *for* neighburhood *read* neighbourhood.
—— 139, —— 21, *for* polaris = $0^h\ 54'37''$ *read* α polaris . . . $0^h\ 54^m\ 37^s$
—— 158, —— 17, *for* a second, *read* half a second.
—— 177, —— 15, *for* 20 years, *read* 30 years
—— 257, —— 14, *for* Pb *read* Px
—— 258, —— 20, *for* (art 116) *read* (art 110)
—— 260, —— 9, *for* r' *read* p'
—— —— 10, *for* $r't$ *read* rt'
—— 262, —— 15, *for* sin *read* sin SP
—— 263, —— 18, *for* rZP *read* ZrP.
—— 265, —— 26, *for* 133, *read* 176

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INTRODUCTION.

The science of astronomy has advanced to its present state, by means of a series of observations and discoveries made during a long course of ages. We can now select from these, such as will best conduce to demonstrate the true system, and explain the various phenomena

Astronomy, by making known to us the immensity of the creation, necessarily increases our reverence of the Divine Creator. This alone, is a sufficient reason for making it a part of general education. It also, perhaps, furnishes a more satisfactory application of the abstract sciences, than any other part of Natural Philosophy. Its practical utility is also considerable. It has always been useful in Geography and Navigation, and lately has afforded splendid assistance to the latter, by the lunar method of finding the longitude at sea

When the student first applies himself to subjects of Natural Philosophy, it is of much importance that he should proceed by the same strict and accurate manner of investigation, to which he had been accustomed while engaged in the rudiments of Mathematics.

The perfection of modern instruments, and of modern observations, admits of an arrangement, which will afford, with respect to the most important facts in Astronomy, nearly the same degree of conviction to the mind as it receives from the elements of Euclid, and which requires little more preparatory knowledge. Such an arrangement has here been had in view.

The phenomena of the celestial bodies observed by a spectator fixed in one place are noticed. The uniform apparent diurnal motion of the concave surface carrying with it the sun, moon, planets and fixed stars, leads to the definitions of the celestial equator, poles, meridian, declination, &c. The considerations of the apparent motions of the sun, moon and planets, on the apparent concave surface, lead to the definitions of the ecliptic, of right ascension, longitude, &c. The various problems of the sphere have their origin from the apparent motion of the concave surface and the apparent motions of the sun, moon and planets on this surface. This is almost all the astronomical knowledge that could be attained to by a spectator fixed to one spot, and not possessing observations made in distant places. He could form no accurate notions of the actual magnitudes, and actual distances of the sun, moon and planets. All the certain astronomical knowledge that existed for many ages was limited to the doctrine of the sphere.

The next consideration is, in what manner we can ascertain the actual magnitudes of the celestial bodies and their

actual distances from us. Telescopes enable us to examine more exactly their appearances, and serve to exhibit many most interesting phenomena, but do not directly lead us further.

The first step of importance is a knowledge of the form and magnitude of the earth. The fixed stars appear in the same relative situations, at the same angular distances from each other, and from the visible celestial pole, in whatever part of the earth we are. The most exquisite instruments point out no alteration. The conclusion drawn from this is, that the fixed stars are at distances so great, that lines directed from all places on the surface of the earth towards the same fixed star, or towards the visible celestial pole, must be considered as parallel. Combining this with what has been observed in so many places, that the variation of altitude of the celestial pole is proportional to the space gone over in a direction north or south, and that for a change of altitude of one degree, the space is about 69½ miles, it is easily proved that the earth is nearly a sphere of about 8000 miles in diameter.

This is an important step—We thus ascertain that a space of 8000 miles is as nothing compared with the distances of the fixed stars.

It also follows that the altitude of the celestial pole is equal to the latitude of the place. This conclusion enables us to solve the problems arising from the situation of the celestial circles in different places, and to explain the variety

of seasons over the whole earth, independent of the knowledge of the true system.

Having ascertained the form and magnitude of the Earth, the next step is to investigate the magnitudes of the sun and planets, or at least to show, that some of them greatly exceed the earth in magnitude, and also to show the vast distances of them compared with the diameter of the earth. It is important that this should be done previously to demonstrating the true system.

Certain observations, made with micrometers, at two places considerably distant from each other, but nearly under the same meridian, serve for this purpose. The student will readily apprehend this method; he will see, that by it we are enabled to ascertain the angle, the disc of the earth would be seen under, could we remove ourselves to a planet to make the observation. This angle can be ascertained with as great precision, as we can measure the apparent diameter of a planet seen from the earth. If, with respect to some of the planets, the angle which the earth's disc subtends be so small, that it is within the limit of the errors of observation, yet we obtain a limit of the magnitude of the earth compared with the magnitude of the planet. Thus the earth seen from Jupiter subtends an angle of four seconds, when Jupiter seen from the earth subtends an angle of forty seconds Now if it be contended that neither of these angles can be ascertained to a second or two, it will make no difference as to the purpose, for which this mode

of ascertaining the relative magnitudes of the earth and Jupiter is introduced. It will sufficiently show that the magnitude of Jupiter greatly exceeds that of the earth, and also will show, that the distance of Jupiter is many thousand times greater than the diameter of the earth

The spots upon the sun, and appearances in several of the planets, show that they are spherical bodies, having a motion of rotation on their axes. All this, being quite independent of any hypothesis as to the arrangement of these bodies, assists much in the arguments by which the rotation of the earth on its axis, and its annual motion round the sun in an orbit nearly circular, may be proved.

The different motions of the planets on the concave surface which appear so irregular, are easily explained by their moving in orbits nearly circular about the sun.

By following an arrangement of this kind, any student may without difficulty satisfy himself of the truth of the Copernican system. He will find this manner of treating the subject pursued in the first seven chapters of this work. At the end of the seventh chapter is a short account of the Ptolemaic System, now no longer interesting, except on account of the ingenuity exhibited in accommodating it to the different phenomena.

After the true system has been explained, the subsequent arrangement in a treatise on astronomy seems of little consequence.

In this work, after the motions of the primary planets

are explained in a general manner, the motions of the moon and secondary planets and several other circumstances connected therewith are briefly noticed. This is followed by some considerations respecting the solar system and fixed stars

A short account of instruments and observations, by which the places and motions of the celestial bodies are exactly ascertained, is followed by a more exact statement of the planetary motions, and by an account of Kepler's discoveries; also by a more particular account of the motions of the moon, of the satellites and of comets.

Several of the phenomena, which arise from, or are pointed out by, the motions and bodies of the solar system, are next considered. Such are the eclipses of the Sun and Moon, the transits of Venus and Mercury over the Sun's disc, the velocity and aberration of light, and the equation of time.

The application of astronomy to navigation and geography is also introduced, and the importance of the former has occasioned a rather long detail.

The chapter on the discoveries in physical astronomy contains little more than an historical account. It had been at first intended that it should contain the elementary parts of physical astronomy, as far as respected Kepler's discoveries. Physical and plane astronomy are now so connected that it is difficult to treat of them separately.

Facts in the history of astronomy have been only occa-

sionally introduced. The student, who has made himself so well acquainted with astronomy as to find its history interesting, will easily procure for himself, from a variety of authors, all the information he can desire.

Among the various advantages derived from the science of astronomy, there is one eminently deserving of notice. We see the most complex appearances and most intricate apparent motions admitting of the simplest explanations.

How intricate and various are the apparent motions which depend only on a primary motion of projection and the simple law of gravity! This may assist us in other departments of natural science, and may encourage us to expect that the most difficult phenomena may at last be found to arise from the most simple laws.

The aberration of light furnishes a remarkable illustration.

Light moves about 200,000 miles in a second, had it moved only 50 miles in a second, it is probable astronomy would not now have existed as a science The motions of the stars and planets would have appeared inextricable confusion. The face of the heavens would have been continually changing, and could not have been divided into constellations. Stars which at one time would be seen close together, at another would appear many degrees asunder. All this would be occasioned by the simple change of the velocity of light, and, as is easily understood, would arise

from a combination of the motion of light, and of the other motions in the system. If this notion be pursued in all its bearings, it cannot be doubted that a consequence of such an alteration in the velocity of light would have been, that this science, by which our knowledge of the creation is so much extended, would scarcely as yet have existed.

ELEMENTS

OF

ASTRONOMY.

CHAPTER I.

ON THE DOCTRINE OF THE SPHERE.

1. The imaginary concave surface in which a spectator at first conceives all the heavenly bodies placed, is an hemisphere, in the centre of the base of which he himself is situate. The base of this hemisphere is the plane by which his view of the heavens is bounded. It is called the *plane of the horizon.*

The numerous bodies observed on the concave surface differ in lustre, and apparently in magnitude. All of them appear to have a daily motion. Many of them emerge, as it were, from below the plane of the horizon, and, after traversing the concave surface, disappear, to rise again at the same points of the horizon as before. Others in their paths never reach the horizon, but continually move round a fixed point in the heavens.

Far the greater number of the celestial bodies preserve the same situation with respect to each other; that is, they preserve the same apparent distances from each other. These are called *fixed stars.*

The sun, besides his diurnal motion of rising and descending, seems also to have a motion on the concave surface, and in a certain space of time, called a year, to return to the same position with respect to the fixed stars

The moon appears also, besides its diurnal motion, to have a motion among the fixed stars, and in a space of time called a month, returns nearly to the same position with respect to the sun.

2 The spectator viewing those stars that do not set, will observe one of them nearly immoveable This is called the Polar Star, from its vicinity to the point about which the stars that do not set appear to move. The point itself is called the *North pole.*—The face of the spectator being turned to this point, the stars rise on his right hand, or in the east, and set on his left hand, or in the west; and thus the apparent diurnal motion of the celestial bodies that rise and set, is from east to west

The apparent motions of the sun and moon among the fixed stars, are in a contrary direction, that is, from west to east

Besides the sun and moon, and fixed stars, ten other celestial bodies may be noticed, which, beside their apparent diurnal motions, have apparent motions that at first seem not easily brought under any general laws Sometimes they appear to move in the same direction among the fixed stars as the sun and moon; at other times in a contrary direction, and then are said to be retrograde. At times they appear nearly stationary. They are called planets They have been named Mercury, Venus, Mars, Ceres, Pallas, Juno, Vesta, Jupiter, Saturn, and the Georgium Sidus Of these, five have been noticed from the remotest antiquity The other five, lately discovered, are only visible by the assistance of telescopes. The Georgium Sidus was discovered by Dr. Herschel in 1780. Ceres was discovered on the first day of the present century, at Palermo, in Sicily, by M. Piazzi. The other three have been discovered

since: Pallas, at Bremen, by Dr. Olbers; Juno, at Lilienthal, by M. Harding; and Vesta, by Dr. Olbers. Mercury and Venus are remarked to be never far from the sun. All but Pallas are always found to be near the annual path of the sun in the concave surface.

3. The above are a few of the phenomena which offer themselves in contemplating the heavens. But the motions are in general only apparent, and take place from a combination of a variety of different motions. The difficulty of deducing the actual circumstances of the magnitudes, and of distinguishing the true from the apparent motions of these bodies, however easy it may appear when done, is such that we ought not to be surprised that the ancients made so little progress toward the knowledge of the true system and true dimensions of the universe; nor ought we to think lightly of their efforts, and to treat them with contempt for their errors. The moderns, by the joint assistance of mechanics, optics, and mathematics, have advanced the science of astronomy to a greater degree of prefection, perhaps, than any other branch of natural knowledge.

4. For more readily explaining and referring to the phenomena of the celestial bodies, certain circles are imagined to be described on the concave surface. Distances on the concave surface are measured by arches of great circles. The[a] present division of the circle into 360 equal parts, called degrees, of each degree into 60 equal parts, called minutes, and of each minute into 60 equal parts, called seconds, was not used till long after astronomy had attained to a considerable degree of perfection.

It is much to be regretted, that, at the revival of learning in Europe, a decimal division of the circle was not adopted, which

[a] The old mode of expressing the measure of an arch, was by stating its relation to the whole circumference; thus the diameter of the sun, measured on the concave surface, was said to be $\frac{1}{720}$ of a great circle.

would have greatly facilitated astronomical computations. The French have lately adopted this division, but not generally.—They divide the circle into 400 parts, each quadrant containing 100, each of these parts into 100, &c. But it is much to be doubted whether the advantages of this division will compensate for the disadvantages now attending it, which necessarily arise from the number of books in which the old division is used, and the great variety of measures of that division familiar to astronomers. Accordingly, in France the old divisions seem likely to prevail, and much inconvenience will probably result from the new divisions having been adopted in some recent very valuable works in astronomy.[a]

The circles, and arches of circles, forming parts of the instruments used in practical astronomy, are actually divided into degrees and parts of a degree, as far as the magnitude of the radius will permit, so that the divisions may not be too close together. The arches or limbs of the largest astronomical quadrants and circles are divided into intervals of 5 minutes. But the measure of an angle can be obtained with great precision by the assistance of ingenious contrivances, that will be noticed hereafter. The most improved instruments are thus adapted to measure angles to seconds. In general, with the best instruments, the result of a single observation can now be depended on to a very few seconds, and in many cases to one second.

5. Let us return to the consideration of the visible concave surface of the heavens. The intersection of the plane of the horizon, with the imaginary concave surface, is a great circle,

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[a] M. Laplace, in his great work, entitled "Mecanique celeste," uses the decimal division. In the new tables of the sun and moon, published by the Board of Longitude in France, 1806, the sexagesimal division is used. In the tables of Jupiter and Saturn, since published by them, the decimal division is used. These are among the most important astronomical publications that have ever appeared.

which may be called the *celestial horizon* A plumb line hanging freely and at rest, is perpendicular to the plane of the horizon, and a small fluid surface at rest is in the plane of the horizon These two circumstances are of the utmost importance to the practical astronomer The impossibility of having, except at sea, an uninterrupted view, and other causes, make it difficult for him to use the horizon itself, but the plumb line and fluid surface fully compensate for these inconveniences.

The *altitude* of a celestial object is its distance from the horizon, measured, on a great circle passing through the object, and at right angles to the horizon. Such a circle is called a secondary to the horizon, a great circle at right angles to another great circle, being called a *secondary* circle. And the *zenith distance* of a celestial object is its distance from the upper pole of the horizon, which is called the *zenith*. By the assistance of a plumb line and quadrant, the altitude or zenith distance may be readily found. Let ACQ (Fig 1) be an astronomical quadrant, the arch AQ of which is divided into degrees, &c., the radius AC is adjusted perpendicular to the horizon, by turning the quadrant about the point C, till a plumb line, suspended from C, passes over a point A The radius CQ is then horizontal A moveable radius or index CT is placed in the direction CO of the object, by means of plain sights at the extremities of the radius C and T (now rarely used), or by means of a telescope affixed to the radius. The arch TQ will then shew the altitude, for TCQ equals HCO the altitude; and the arch TA will shew the zenith distance, for ACT equals OCZ the zenith distance. The method of observing altitudes will be more accurately described hereafter: it was thought necessary to advert to it here; and also to mention how an angular distance on the concave surface may be measured.

A circle HABG (Fig. 2) divided into degrees, &c., furnished with a fixed radius AC, and a moveable radius BC, be-

ing placed in the plane passing through two objects and the eye, the circle may be turned till the fixed radius AC passes through one object, and then the moveable radius BC being made to pass through the other, the arch AB will shew the angular distance This method is now rarely used The angular distance of two objects when required, is seldom *directly* observed, on account of the inconvenience of adjusting the plane of the instrument, and the two radii, to two objects, both of which perhaps are moving, and with different motions. Therefore, in this way, great accuracy cannot be attained · but the conception of this method, although inaccurate, will be useful in what follows When an angular distance on the concave surface is required, it is generally obtained by computation from other observations, *e g* from the declinations and right ascensions (to be explained hereafter). In one instance, indeed, *in the lunar method of finding the longitude*, it is necessary to observe, with great precision, the distance of the moon from the sun or a fixed star This is done in a manner hereafter described, by an Hadley's sextant, an invaluable instrument for the purpose. By this instrument also the angular distance between any two objects may be measured

6 To explain the phenomena of the apparent diurnal motions of the celestial bodies, we imagine an hemisphere below our horizon, and in it a point diametrically *opposite to the north* pole, which we call the *south celestial pole*; we also imagine that the concave surface turns uniformly on an axis, called the axis of the world, passing through the north and south poles, completing its revolution in the space of $23^h\ 56^m$ nearly, carrying with it the sun, moon, and stars, while the horizon remains *at rest*.

This hypothesis illustrates and represents the apparent diurnal motion of the several celestial objects in parallel circles, with an equable motion, each completing its circular path in the same

time. That the motion of each star is equable, and that they describe parallel circles on the concave surface, we deduce from observation and the computations of spherical trigonometry. This will be readily understood from what follows

The great circle, the plane of which is at right angles to the axis of the world, is called the *Equator*. This circle is bisected by the horizon, and therefore all celestial bodies situated in it are, during equal times, above and below the horizon, consequently when the sun is in this circle, day and night are of equal length, whence it is also called the *equinoctial*

This representation of the diurnal motion, by the motion of a sphere about an axis inclined to a plane representing the horizon, on which sphere the celestial bodies are placed at their proper angular distances, must have been among the first steps in astronomy Yet in the infancy of the science, doubtless, a considerable time elapsed before it was known that the diurnal paths of the stars were parallel circles, described with an equable motion. Without this, little progress indeed could have been made It is likely that at first it was little more than an hypothesis, in some degree confirmed by the construction of a sphere, to represent by its motion the celestial diurnal motions; for its confirmation, by the application of spherical trigonometry, seems to require a greater knowledge than we can suppose then existed.

This diurnal motion, we now know, is only apparent, and arises from the rotation of the earth about an axis, by which the horizon of the spectator revolves, successively uncovering, as it were, the celestial bodies, while the circles of the sphere are at rest But the phenomena are the same, whether the horizon is at rest and the imaginary sphere revolves, or the horizon revolves and the imaginary sphere is at rest By conceiving the sphere to revolve and the horizon to be at rest, the phenomena are more easily represented. Three centuries since, this appa-

rent diurnal motion was generally considered to be real; and had we not the knowledge derived from navigation, and the communication of observations made in distant countries, we might still contend for the truth of it Now we only imagine it, for more readily explaining the phenomena of the sphere and the circles thereof.

7. *Circles of the sphere.*—Secondaries[a] to the equator are called *circles of declination,* because the arc of the secondary, intercepted between an object and the equator, is called its *declination north* or *south,* according as the object is on the north or south side of the equator.

The great circle passing through the pole and the zenith, is called the *meridian.* This circle is at right angles, or a secondary, both to the horizon and equator It is easy to see that it divides the visible concave surface into two parts, eastern and western, in every respect similarly situate as to the pole and parallel circles The eastern parts of the parallel diurnal circles being equal to the western, and the motions equable, the times of ascent from the horizon to the meridian,[b] are equal to the times of descent from the meridian to the horizon

In (Fig 3) the circle HFKOGW represents the horizon, the centre C of which is the place of the spectator. The part

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[a] A common celestial globe, or even a reference to the concave surface itself, will much better assist the conception of the circles of the sphere, than figures drawn on a plane surface, which are rather apt to mislead a beginner. The horizon of the globe must be considered as continued to pass through the centre, where the eye is supposed situate viewing the hemisphere above the horizon, and the axis of the globe is to be placed at the same elevation, as the axis of the concave surface of the spectator In this way all the circles of the celestial sphere will be easily understood Any consideration of the form of the earth is entirely foreign to a knowledge of the circles of the sphere They were originally invented without any reference to or knowledge of it.

[b] Vide Appendix, Prop I

of the figure above this circle represents the visible concave surface, and the part below, the invisible. Z is the zenith, P the visible, and R the invisible pole. PZEHRNQO is the meridian, EGQV the equator. AB a small circle parallel to the equator, FLW the visible portion of another parallel to the equator. A star, situate in AB, is continually above the horizon. A star in the equator is only visible while in the part GEV equal to VQG. A star in FLW is only visible in the portion FLW above the horizon, it rises at W, and sets at F. ZSK is a portion of a secondary to the horizon. SK is the altitude of the point S, and SZ its zenith distance. PSD is a secondary to the equator, or a circle of declination, and DS the declination of the point S.

A telescope being directed to any star, and the time noted by a clock, if the telescope remain fixed, the same star will again pass through it after an interval of $23^h\ 56^m$ nearly. And the time of passing over the aperture of the telescope being the same to whatever part of the star's diurnal path the telescope is directed, proves the equable motion in that diurnal path. A telescope particularly fitted up, and placed so as to be conveniently moved in the plane of the meridian, is of as much use in the practice of astronomy as the quadrant: it is called a transit instrument; its uses will be afterwards explained, as well as the method of finding the direction of the meridian.

The time of describing a diurnal circle by a star may be nearly ascertained, without a telescope, by suspending two plumb lines at two or three feet from each other, then observing when the star appears in the plane of the strings, noting the time by a clock well regulated: the same star will pass the plane again after $23^h\ 56^m$. An upright wall will serve for the same purpose. Vice versâ, this method will serve to ascertain the rate of going of a clock. It may also be applied to ascertain the time of passage over the meridian, by adjusting the plumb lines in the plane of the meridian.

Secondaries of the equator are also called *hour circles*, because the arc of the equator, contained between any one of these circles and the meridian, shews the distance in time of that body from the meridian, the equator being divided into 24 hours

8. The meridian also passes through the *nadir* (the lower pole of the horizon)

Secondaries of the horizon are called *vertical circles*. That vertical circle which intersects the meridian at right angles is called the *prime vertical*.

It will help the conception of the student to consider the meridian and other verticals of the horizon as remaining at rest, while the sphere revolves, carrying the equator and other circles

The four points where the meridian and prime vertical intersect the horizon, are called the cardinal points Those of the meridian, north and south, those of the prime vertical, east and west. The equator intersects the horizon in the east and west points (being poles of the meridian), and its inclination to the horizon equals the complement of the altitude of the celestial pole. The prime vertical also intersects the equator at the east and west points, and at an angle equal to the altitude of the pole.

The *azimuth*[a] of a celestial object is measured by an arc of the horizon, intercepted between the meridian and a vertical circle passing through the object. In (Fig 3) KO is the azimuth of the point S from the north.

The altitude of a celestial object, being its distance from the horizon measured on a secondary of the horizon, is greatest when the object is on the meridian

9. The path of the Sun traced on the *surface of the imaginary* celestial sphere, among the fixed stars, is a great circle,

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[a] The complement of the azimuth, or the arc intercepted between the prime vertical and the vertical through the object, is called the *amplitude* Ed.

which he moves over, in a direction from west to east. This circle is called the *ecliptic*, because eclipses take place when the *moon*, at the new and full, is *in or near this circle*. The apparent motion of the sun, in this circle, is not entirely uniform; the motion being contrary to the *diurnal* motion, the interval between two meridian passages of the sun is greater than that of the fixed stars, and by four minutes nearly. This interval, between two passages of the sun over the meridian, is in its mean quantity called 24 hours, or a day. In 365 days, 6 hours and 9 minutes, the sun appears to complete the ecliptic. The seasons are connected with the positions of the sun in the ecliptic. The period, therefore, of his motion, called a year, becomes one of the most important divisions of time.

10. The moon completes her course among the fixed stars, by a motion from west to east, in 27 days 7 hours, returning nearly to the same place. Its apparent path is *nearly* a great circle, intersecting the ecliptic at an angle of about five degrees. Its motion also being contrary to the diurnal motion, the interval between its successive passages or transits over the meridian is greater than that of the fixed stars, and by 52 minutes, in its mean quantity. The moon is said to be in opposition to the sun, when near that part of the ecliptic opposite to the sun. The interval between two oppositions is nearly 30 days, and at each opposition the moon shines with a full phase. The use, in civil life, of this striking phenomenon, makes another important division of time, which is called a *month*.

11. The ecliptic necessarily intersects the equator, each being a great circle. The angle of intersection is nearly 23° 28′. The circumstance of the inclination, or obliquity of the ecliptic to the equator, explains the change of seasons. The true cause of the appearance of the obliquity of the ecliptic to the equator, will be afterwards shewn. If the ecliptic coincided with the equator, the sun would always rise and set in the east and west points, would always be at the same altitude when on

the meridian, and would be absent and present during equal spaces of time. Now the effect of the sun, with respect to heat, depends upon the time of his continuance above the horizon and the greatest altitude to which he rises; therefore, if he moved in the equator, no alteration would take place, because these would be the same every day. But the ecliptic being inclined to the equator, when the sun is in that part which is between our visible pole and the equator, the greater part of each of the diurnal circles which he describes, is above our horizon, i. e. he is more than half the 24 hours above the horizon, and he passes the meridian between the equator and zenith. When southward of the equator, he is less than 12 hours above the horizon. When he is in the points of intersection of the ecliptic and equator, he is just 12 hours above the horizon, and it is then equal day and night. This latter circumstance takes place on the 20th of March and 23rd of September.

The sun is in that part of the ecliptic nearest our visible pole about the 21st of June, and then our days are longest, and in the part farthest from it on the 21st of December, when our days are shortest. The sun is about eight days longer on the northern side of the ecliptic than on the southern, and hence summer is eight days longer than winter. The greatest heat is not when the days are longest, but some time after, because the increase of heat during the day is greater than the decrease during the night, consequently heat must accumulate till the increments and decrements are equal; afterwards the decrease being greater than the increase, the heat will diminish. The same may be said with respect to cold.

12. The two parallels to the equator, or parallels of declination, touching the ecliptic, are called *tropics* or *tropical circles,* because when the sun is in these points of the ecliptic, he turns his course, as it were, back again toward the equator.

The points of the ecliptic of greatest declination, or the

tropical points, are called *solstices*, because the sun appears stationary, with respect to his approach to the poles.

13. A belt or zone extending on each side of the ecliptic about 8° is called the *zodiac*, from certain imaginary forms of animals conceived to be drawn in it, called *signs* of the zodiac. There are twelve signs, probably from there being twelve lunations during the course of the sun in the ecliptic. These are Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces, and denoted by ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓. The reason of distinguishing this space was, because the sun and planets were always observed within it. These figures served also to distinguish the position of the stars with respect to one another, and were therefore called the *constellations* of the zodiac. The space of the zodiac has always been noticed from the earliest records of astronomy. Some of the planets lately discovered are not confined to this space. One of them, Pallas, sometimes is distant above 62° from the ecliptic.

The first six constellations, beginning with Aries, were formerly on the northern side of the ecliptic, most probably when the description of the zodiac was first invented, and the six others on the southern. But by a comparison of observations made at a considerable interval from each other, it is found that the intersections of the ecliptic and equator move backward, in respect to the signs of the zodiac, the obliquity of the ecliptic remaining nearly the same. The equator moves on the ecliptic, the ecliptic continuing to pass nearly through the same stars. The intersections or the equinoctial points move backward at the rate of 1° in 71½ years, and therefore, at present, the constellation Aries seems to be moved forward nearly 30° from the equinoctial point, yet astronomers still commence the twelve signs or divisions of the ecliptic at the equinoctial point, and name them after the constellations of the zodiac. This distinction ought to be attended to.

14 In the practice of astronomy, the most general and convenient method of ascertaining the position of any celestial object on the concave surface, is to determine its position with respect to the equator and vernal equinoctial point, that is, to determine its *declination* and *right ascension.* The *right ascension* of a celestial body is the arc of the equator intercepted, (reckoning according to the order of the signs), between the vernal equinoctial point, or the first point of Aries, and a secondary to the equator passing through the object This is expressed both in time and space. Thus, if the arc intercepted be 15°, the right ascension may be said to be 15° or one hour, supposing the equator divided into twenty-four hours. The measure of twenty-four hours for the time of the diurnal revolution of the fixed stars, or the celestial sphere, is called *sidereal time* Hence the interval in sidereal time between the passages of two fixed stars over the meridian, is the same as the difference of their right ascensions expressed in time

The term, right ascension, originally had a reference to the rising of the celestial bodies Now its use is much more circumscribed, but much more important, and therefore it might have been better to have adopted another term for expressing the arc intercepted between a secondary to the equator passing through the celestial object, and the first point of the equator

15 The position of a celestial body, with respect to the equator, being ascertained, it is very often necessary to ascertain its position with respect to the ecliptic, i. e. to determine its *longitude and latitude.* This is done by spherical trigonometry.[a]

The *longitude* of a celestial object is measured by an arc of the ecliptic, intercepted between the first point of Aries

[a] Vide Appendix, Prop. IV

(reckoning according to the order of the signs) and a secondary to the ecliptic passing through the object. Its *latitude* is its distance from the ecliptic, measured on a secondary of the ecliptic passing through the object.

16. The *solstitial colure* is a secondary to the equator passing through the solstices, and is therefore also a secondary to the ecliptic.

The *equinoctial colure* is a secondary to the equator, passing through the equinoctial points.

CHAPTER II.

FIXED STARS—TELESCOPES—APPEARANCE OF STARS IN TELESCOPES.

17. Let us now return to the consideration of the fixed stars We observe about 3000 stars visible to the naked eye, very irregularly scattered over the concave surface of the heavens There are seldom above 2000 visible *at once*, even on the most starry night They are distinguished from the planets not only by preserving the same relative position to each other, but also by a tremulous motion or twinkling in their light, apparently arising from the effect of the atmosphere on the rays of light passing through it

For the conveniency of arranging and referring to the different stars, the method of constellations was invented by the ancients. They imagined a number of personages of their mythology, also animals, &c drawn on the concave surface, and including particular groups of stars; these they called constellations, and denominated the stars from the constellation in which they were, and from their situation in that constellation. This method, though certainly useful, is not adequate to the purposes of astronomy in its present state, but for many obvious reasons it has been retained. The stars do not form the figure of the constellation, except in a few assemblages which have a remote resemblance, such are the Great Bear, the Hyades composing the Bull's head, &c. Some of the brighter fixed stars, and those more remarkable by their position, had proper names assigned to them, as Arcturus, Sirius, Alioth, Algol, &c.

18. Bayer, who published a celestial atlas in the year 1603, much facilitated the arrangement of the fixed stars. He marked those in each constellation by the letters of the Greek alphabet, according to their then degrees of brightness.[a] The stars are also divided according to their apparent brightness into magnitudes. The brightest are of the first magnitude, and so on to the sixth, the least magnitude visible to the naked eye. There are eleven stars of the first magnitude in the portion of the concave surface visible to us, viz., Aldebaran, Capella, Rigel, α Orionis, Sirius, Regulus, Spica Virginis, Arcturus, Antares, α Lyræ, and Fomalhaut. In the remaining portion of the concave surface there are six, viz., Achernar, Canopus, β Argûs, α Crucis, α and β Centauri. There are about 50 of the second magnitude, and about 120 of the third magnitude, visible to us.

Some assemblages of the stars are more remarkable than others, such are the Pleiades, Hyades, Cassiopæa's chair, and the great Bear. The eye unassisted by a telescope, remarks also a very considerable irregular luminous belt called the milky way. Likewise other small luminous spots, called from their appearance nebulæ, viz. Præsepe, nebulæ in Perseus, in the girdle of Andromeda, &c. By the assistance of telescopes, we find that the number of the fixed stars is greater than can be ascertained; the number of those visible to the naked eye being incomparably smaller than of those which are only visible by the help of telescopes.

19. The theory of telescopes properly belongs to the science of optics, and therefore a very short account of their effects, and of the improvements that have, from time to time, been made in them, is all that is necessary here.

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[a] The constellation called the Great Bear is an exception, in it the principal stars are marked in the order of their right ascensions.

The use of telescopes is to magnify objects, or to present their images under a larger angle than the objects themselves subtend; and likewise to render objects visible that would otherwise be invisible. Telescopes for common astronomical purposes magnify from 40 to 200 times, and for particular purposes from 200 to 1000 and upwards; i. e. objects appear so much nearer than when seen by the naked eye, and their parts become more visible and distinct. We are enabled by a telescope which magnifies 100 times to behold the moon the same as we should if placed 100 times nearer than at present. A telescope magnifying a thousand times, will exhibit the moon as we should behold it could we approach within 240 miles of it. Thus, although we cannot actually approach the moon at pleasure, we can form an image of the moon, and approach this image at pleasure, and so make the image subtend a greater angle than the moon itself. We can magnify the image by help of a simple microscope, as we can magnify any minute object. This is the principle of the common telescope. The object glass forms an image of the moon, and we magnify this image by help of the eye glass, which may be considered as a microscope.

20. Telescopes were accidentally invented at Middleburgh, in Holland, about the year 1609. There is no foundation for supposing them known earlier, although the single lens had been in use for spectacle glasses since the beginning of the 14th century. Galileo, hearing of their effects, soon discovered their construction, and applied them to astronomical purposes, from whence a new æra may be dated in astronomy. After some trials, Galileo made a telescope which magnified upward of 30 times, and with this instrument, so inferior in power to modern telescopes, he made most important discoveries. In little more than a year he had observed the nebula of Orion, the telescopic stars in the Pleiades and in Præsepe, had discovered the satellites

of Jupiter, very accurately described the face of the moon, and computed the height of some lunar mountains, observed an extraordinary appearance in Saturn, occasioned by the ring, which his telescope could not clearly shew, and had observed phases in Venus similar to the phases of the moon.

Notwithstanding the importance of the telescope, it was but slowly improved. Telescopes admitting of a high magnifying power were of a very inconvenient length. A high magnifying power could not be obtained by a short telescope, without rendering the image indistinct by colour. The ardour and industry of the astronomers of the latter end of the 17th century overcame this difficulty, by using telescopes without tubes. They affixed the object glass to the top of a pole, directing it by means of a long string, so as to throw the image into its proper place. Huyghens particularly distinguished himself by important discoveries with this inconvenient kind of telescope, which has been called the *aerial telescope*. The discoveries of Sir Isaac Newton, respecting light, induced astronomers to desist from endeavouring to improve refracting telescopes, and to aim at perfecting reflecting ones. Soon after the discovery of the telescope, it was suggested that the image of the object might be formed by reflection instead of refraction, but as no particular advantage could be shewn to arise from this alteration, it does not seem to have been attended to, till James Gregory proposed the construction of a reflecting telescope which goes by his name. He intended by this construction to obviate the errors of the object glasses of the common telescope, arising from their being necessarily ground of a spherical form. The discoveries of Newton on light shewed these errors to be comparatively of trifling consequence. Newton himself, as soon as his experiments on light had shewn him the true obstacle to the improvement of refracting telescopes, invented and executed a reflecting telescope, which goes by his name. His construction

is better adapted to many purposes in astronomy than that of Gregory, although for common purposes Gregory's may be considered most convenient

Many inconveniences attended the construction and execution of reflecting telescopes When made, they were liable to tarnish, and to change their figure, an error in which is of much greater consequence than in refractors. Thus much fewer advantages were derived from reflecting telescopes than had been expected And the improvement of telescopes seemed at a stand, when, in the year 1757, a discovery of Mr Dolland, an optician in London, gave hopes of improving them far beyond what had been hitherto done He discovered that by a combination of lenses of flint glass and crown glass, he could form an image free from colour This enabled him to make telescopes, admitting of high magnifying powers, of a very convenient length These telescopes, called *achromatic*, are now in common use, and fitted to those astronomical instruments by which angles are measured. Expectations were formed of being able to increase the breadth of the object glasses, to admit of very high magnifying powers, without lengthening the telescope so as to be inconvenient, but this was prevented by the nature of flint glass This cannot be obtained fit for the purpose of telescopes, except in small portions of surface. It does not appear that we can do more by achromatic telescopes, than astronomers at the end of the 17th century did by telescopes without tubes, if so much, and achromatic telescopes, although an invaluable improvement by reducing the length of telescopes, have not discovered to us more in the heavens than had been seen a century before

21 Under these circumstances, the very ingenious and indefatigable Dr Herschel set himself to improve reflecting telescopes, in which he has been highly successful His reflectors are of the Newtonian kind. After repeated attempts he suc-

ceeded in making one 20 feet long and 18 inches aperture The great breadth of the aperture increased so much the brightness of the image, that he was enabled, with great convenience, to use very high magnifying powers At last he attempted and executed one 40 feet in length and of 4 feet aperture A most surprising performance, when the labour and difficulty of casting and polishing the metal speculum, the obstacles he had to contend with in the weight, and in the apparatus for moving it, are considered. A full account of this telescope, by Dr. Herschel himself, is given in the Phil Trans. for 1795

The discoveries of Dr Herschel will be mentioned in their places. In the mean time it may be remarked, in order to form some idea of the effect of telescopes, when applied to the celestial bodies, that the reflector of the 40 feet telescope forms an image of the ring of Saturn, about $\frac{1}{10}$ of an inch in diameter; we are enabled to magnify this by the eye glass, in the same manner as we can magnify an object $\frac{1}{10}$ of an inch in breadth by a common microscope.

22 The appearance of the stars seen in a telescope, is very different from that of the planets. The latter are magnified, and shew a visible disc The stars appear with an increased lustre, but with no disc Some of the brighter fixed stars appear most beautiful objects, from the vivid light they exhibit Dr. Herschel tells us, that the brightness of the fixed stars of the first magnitude, when seen in his largest telescope, is too great for the eye to bear. When the star Sirius was about to enter the telescope, the light was equal to that on the approach of sun rise, and upon entering the telescope, the star appeared in all the splendour of the rising sun, so that it was impossible to behold it without pain to the eye

The apparent diameter of a fixed star is only a deception arising from the imperfections of the telescope. The brighter

stars appear sometimes in bad telescopes to subtend an angle of several seconds, and this has led astronomers into mistakes respecting their apparent diameters The more perfect the telescope, the less this irradiation of light We know certainly that some of the brighter fixed stars do not subtend an angle of 1″, from the circumstance of their instantly disappearing, on the approach of the dark edge of the moon Dr. Herschel attempted to measure the diameter of α Lyræ, and imagined it to be about $\frac{3}{10}$ of a second

23 Although the superior light of the sun effaces that of the stars, yet by the assistance of telescopes we can observe the brighter stars at any time of the day The aperture of the telescope collects the light of the star, so that the light received by the eye is greater than when the eye is unassisted. The darkness in the tube of the telescope also in some measure assists [a]

The most inferior telescope will discover stars that escape the unassisted sight By the telescope we discover that the milky way, and some of the nebulæ above-mentioned, consist of very numerous small stars. Others, even in the best telescopes,

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[a] It appears by the principles of optics, that when an object is seen through a telescope, the density of the light on the retina must be always less than when the object is seen by the naked eye, but the quantity of light in the whole image may be much greater in the former case than in the latter. And it is certain that our power of seeing the object with distinctness, depends on the quantity of light in the whole image Dr Herschel, in a valuable paper in the Phil. Trans. 1800, *part I* on the power of penetrating into space, uses the terms *absolute brightness* and *intrinsic brightness*, the former to distinguish the whole quantity of light in the image on the retina, and the latter to distinguish its density. He gives an instance in which the absolute brightness was increased 1500 times in a telescope, and the intrinsic brightness was less than to the naked eye in the proportion of 3 to 7

appear still as small luminous clouds There is a very remarkable one in the constellation of Orion, which the best telescopes show as a spot uniformly bright. It is a singular and beautiful phenomenon So great is the number of telescopic stars in some parts of the milky way, that Dr. Herschel observed 588 stars in his telescope at the same time, and they continued equally numerous for a quarter of an hour In a space about 10 degrees long, and 2½ degrees wide, he computed there were 258000 stars. Phil Trans. 1795

24. The most ancient catalogue of the fixed stars is that of Hipparchus, who observed at Alexandria about 150 B C. His catalogue consists of 1022 stars. Although several celebrated astronomers, as Tycho Brahe, &c. employed themselves in more accurately observing the places of the fixed stars, yet the number was not much increased till the time of Flamstead, whose catalogue, entiled the British Catalogue, appeared in 1725. It contains about 3000 stars visible to the naked eye, and was the result of nearly 40 years labour Later astronomers have observed, with greater accuracy, the places of some of these stars, particularly of those in and near the zodiac; and very recently, M Piazzi, of the Observatory at Palermo, in Sicily, has recompleted the whole catalogue. In 1802, M Delalande published a work entitled *Histoire céleste Française*, in which are observations of 50000 stars, viz. of stars of the 6th magnitude not observed by Flamstead, and of telescopic stars of the 7th, 8th, and 9th magnitudes They were mostly observed by his nephew, M. Lefrançais Delalande, and furnish a lasting monument of his patience and industry Great as is this number of stars of the above magnitudes, it would not be difficult to increase it considerably.

25 Some stars appearing single to the naked eye, when examined with a telescope appear double or treble, that is, consisting of two or three stars very close together : such are Castor,

α Herculis, the Pole Star, &c. Seven hundred, not noticed before, have been observed by Dr. Herschel. They are particularly useful for trying and comparing the goodness of telescopes, because if the telescope do not give a well defined image, these stars will appear as one. In viewing these double stars a singular phenomenon discovers itself, first noticed by Dr. Herschel; some of the double stars are of different colours, which, as the images are so near each other, cannot arise from any defect in the telescope. α Herculis is double, the larger red, the smaller blue; ϵ Lyræ is composed of four stars, three white and one red; γ Andromedæ is double, the larger reddish white, the smaller a fine sky blue. Some single stars evidently differ in their colour. Aldebaran is red, Sirius brilliant white.

From observations at different periods it appears considerable changes have taken place among the fixed stars. Stars have disappeared, and new ones have appeared. The most remarkable new star recorded in history, was that which appeared in 1572, in the chair of Cassiopæa. It was for a time brighter than Venus, and then seen at mid-day; it gradually diminished in lustre, and after sixteen months disappeared. That the circumstances of this star were faithfully recorded we can have no doubt, since many different astronomers of eminence saw and described it. Cornelius Gemma viewed that part of the heavens on November 8, 1572, the sky being very clear, and saw it not. The next night it appeared with a splendor exceeding all the fixed stars, and scarcely less bright than Venus. Its colour was at first white and splendid, afterwards yellow, and in March, 1573, red and fiery like Mars or Aldebaran, in May of a pale livid colour, and then became fainter and fainter till it vanished.

Another new star, little less remarkable, appeared in October, 1604. It exceeded every fixed star in brightness, and

even appeared larger than Jupiter Kepler wrote a dissertation upon it.

Changes have also taken place in the lustre of the permanent stars; β Aquilæ is now considerably less bright than γ. A small star near ζ ursæ majoris is now probably more bright than formerly, from the circumstance of its being named Alcor, an Arabic word which imports sharp-sightedness in the person who could see it It is now very visible.

26. Several stars also change their lustre periodically, *o* Ceti, in a period of 333 days, varies from the 2nd to the 6th magnitude The most striking of all is Algol or β Persei. Mr Goodricke has with great care determined its periodical variations. It is, when brightest, of the 2nd, and, when least, of the 4th magnitude; its period is only 2^d 21^h. it changes from the second to the fourth magnitude in $3\frac{1}{2}$ hours, and back again in the same time, and so remains for the rest of the 2^d 21^h These singular appearances may be explained, by supposing the fixed star to be a body revolving on an axis, having parts of its surface not luminous

27 The number of nebulæ is very considerable. Dr Herschel has discovered above 2000 before his time only 103 were known. But far the greater part of these 2000 can be only seen with telescopes equal to his own. The vast quantity of light obtained by his large speculums, renders his telescopes very useful for discoveries among the fixed stars, for which light is the principal thing to be desired He has given an account of several phenomena, which he calls nebulous stars, stars surrounded with a faint luminous atmosphere. He describes one observed Nov. 13, 1790. "A most singular phenomenon: a "star of the 8th magnitude, with a faint luminous atmosphere, "of a circular form, and of about 3′ diameter; the star is per-"fectly in the centre, and the atmosphere is so diluted, faint, "and equal throughout, that there can be no surmise of its con-

"sisting of stars; nor can there be a doubt of the evident con-
"nection between the atmosphere and the star. Another star, "not much less in brightness, and in the same field with the "above, was perfectly free from any such appearance." Phil. Trans 1791.

Dr Herschel has, with unwearied attention, exerted himself in examining and noting every thing remarkable in every part of the visible celestial surface, by a regular review, so that little can escape him In consequence of his numerous discoveries, many very ingenious and magnificent ideas have occurred to him respecting the fixed stars and nebulæ

28. Having given a short statement of the simple appearances of the bodies placed on the concave surface of the heavens, which are such, that they must strongly excite our curiosity, we may now leave the subject, and resume it after having stated the knowledge that observations and deductions from thence afford us, respecting the magnitudes, distances, and motions of the sun, moon, and planets Then returning again to the consideration of the fixed stars, and assigning them their proper places in the universe, we shall discover what must fill our minds with astonishment and awe, and must raise in us the greatest admiration of the Almighty Creator. That which has hitherto been stated, regards only what a spectator fixed to one spot might discover It is only by a change of place, or by comparing the observations made at places distant from each other, that we can readily arrive at a knowledge of the real distances and real motions of the celestial bodies. An isolated observer, however he might be gratified by the spectacle of the heavens on a fine evening, would be able to discover little of what, when the true circumstances are known, add so much to the wonderful variety we observe in terrestrial matters, of the Creator's power He would only barely discover that the sun, moon, and planets were at different distances from the earth

He would also be able to form hypotheses to explain their motions, but few of those would he be enabled to submit to the test of experience. Previously to this it would be necessary to investigate the figure and dimensions of the earth upon which he lives. This knowledge is obtained from the phenomena which arise from a change of place.

CHAPTER III.

PHENOMENA DEPENDING ON A CHANGE OF PLACE, AND ON THE FIGURE OF THE EARTH.

29 A SPECTATOR, without changing his situation on the earth, would soon discover that the celestial bodies are not all placed on the concave surface at *fixed* distances from him, for he would remark that the sun, moon, and planets varied their apparent magnitudes or diameters, which must arise either from changes of distance, or changes in the actual magnitudes of the bodies The former solution is so much simpler than the latter, that no one could hesitate in adopting it, even if not confirmed by other circumstances Likewise that the heavenly bodies are not placed at *equal* distances from him It was remarked that the apparent paths of the sun and moon intersected each other. When they appear to meet at these intersections, the moon is observed to obscure or eclipse the sun, consequently the moon must be nearer than the sun. But to proceed in the investigation of these distances, it will, as was observed, be necessary to become acquainted with the form of the earth on which we live.

30 A spectator placed on the sea, or on a plain, where his view is unobstructed, at first considers the surface as a plane coinciding with his horizon, and extended to the concave surface of the celestial sphere But it is immediately suggested to him, that the surface of the earth is not flat or coincident with his horizon, for on the sea he perceives the tops of the masts to dis-

appear last, and on the plain he observes the tops of distant objects, when the bottoms are invisible. This cannot be explained otherwise than by a curvature on the earth's surface. The voyages of modern navigators have put this matter in the clearest light; for, by continued sailing to the eastward or westward, they have arrived again at the port from which they set out. This has been done in different courses on the surface, so that thereby traversing the earth, they have ascertained its surface to be a curved surface returning into itself. Eclipses of the moon serve to point out that the figure of the earth must be nearly spherical, for the boundary of the earth's shadow seen on the moon always appears circular, which could not *always* be the case, unless the earth were nearly a sphere.

31. The magnitude of the earth is next to be considered; previously to which it is necessary to remark, that however distant two places on the earth's surface are, the angular distances of the same stars visible in each place are precisely the same; from whence it follows, that the distances of the fixed stars are so great, that each inhabitant of the earth, in respect to them, considers himself in the centre of the same imaginary sphere; or that all lines drawn from the surface of the earth to any star, may be considered as parallel at the surface of the earth; for the inclination of the lines drawn from any two places towards the same star, is less than can be measured, and therefore for all purposes they must be considered as parallel.

32. Every spectator also observes the same celestial pole and equator, that is, situate the same with respect to the fixed stars; but the situation of the celestial circles with respect to the horizon will be different. The meridian altitudes of the celestial objects will be different in different places, and the altitude of the north celestial pole will be increased or diminished, according as an observer travels north or south. Actual admeasurement shews, that the space gone over in a direction

north or south, is very nearly proportional to the variation of the altitude of the celestial pole Measurements shewing this have been made in Lapland, Holland, England, Germany, France, Italy, at the Cape of Good Hope, Hindostan, and in North and South America

33 From hence it is proved that the earth is nearly a sphere, by which is explained the phenomenon of the variation of altitude of the pole, being proportional to the space gone over in a direction north or south

Let the circle LCS (Fig 4) represent a section of the earth, on the plane of a celestial meridian. LR a section of the horizon of the place L, SO of the place S LP and SP′ lines drawn in the direction of the celestial pole, which are therefore parallel, (Art. 31 and 32) Produce SO to meet LR and LP, in H and B Now P′SO = PBO, and PBO − PLR = LHB = C, therefore P′SO − PLR = C But C varies as LS, consequently the difference of the elevations of the pole at L and S varies as LS. Experiment shewing this to be nearly so, it follows that the earth is nearly a sphere It is also proved by navigators, in distant voyages, making their computations of the distances sailed, upon the supposition that the earth is a sphere, and the result nearly agreeing with the distances ascertained by the rate of sailing deduced by the log-line.

34. The measure of a degree on the earth's surface is[a] 69½ British miles nearly, that is, if the difference of PLR and P′SO be 1°, the distance LS = 69½ miles, and therefore 360° or the circumference of the earth = 25000 miles nearly. Hence the diameter, which is somewhat less than ⅓ of the circumference =

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[a] The method of measuring a degree is afterwards explained in the application of astronomy to geography, by which it is found that the earth is not exactly a sphere, its equatoreal diameter being about 25 miles longer than the polar, according to the latest results.

8000 miles nearly. A vast magnitude, when measured by our ideas, but almost nothing when compared with other bodies, the existence of which, in the universe, we are enabled to ascertain

35 It cannot now be determined how long the knowledge of the spherical figure of the earth has existed, but just ideas of it were early entertained. Above 2000 years ago it was commonly known among astronomers. Indeed it must have been discovered in the very infancy of astronomy. It plainly appeared that the eclipses of the moon were occasioned by the intervention of the earth, and the termination of the shadow must soon have pointed out to them the form of the earth. The measure given by Aristotle is the earliest upon record, who reports it from more ancient authors. Eratosthenes, who observed at Alexandria, and died 194 B. C. made use of a method for measuring the earth susceptible of great accuracy. The result of his measurement has come down to us, but from the uncertainty of the length of the stadium used, it has been supposed that we are unable now to appreciate the accuracy of the ancient measurements Although the spherical figure of the earth was universally acknowledged among the astronomers, yet the existence of antipodes was long denied.

36. That diameter of the earth, parallel to the imaginary celestial axis, is called the axis of the earth, and this is properly so called, because, as will be shewn, the earth actually turns upon this axis, thereby causing the apparent diurnal motion of the concave surface.

The great circle of the earth, the plane of which is perpendicular to its axis, is called the *terrestrial equator*. Circles are also conceived to be drawn on the earth, corresponding to the imaginary circles in the heavens. The secondary of the terrestrial equator passing through any place, is called the *terrestrial meridian* of that place. The arc of the meridian intercepted between the place and the equator, is called the *latitude* of the

place, and the arc of the equator intercepted between the meridian of any place and some one given meridian, is called the *longitude* of that place, and is reckoned 180° to the eastward or westward.

37 The British reckon their longitudes from the Observatory of Greenwich, the French from Paris, &c When the Canary Islands were the most westerly lands known, the longitude was reckoned from the meridian of Ferro, one of those islands. The use of the latitude and longitude in fixing the position of a place on the surface of the earth, was first introduced by Hipparchus

It may be remarked here that the progress in astronomy was from the celestial circles to terrestrial, and not the contrary

38. By passing to the southward of the terrestrial equator, we are enabled to behold the part of the celestial sphere near the south pole, which is invisible to us the inhabitants of the northern hemisphere The stars near the south pole have been divided into constellations Dr. Halley and De La Caille went to the Cape of Good Hope, for the express purpose of observing the southern hemisphere

39 The knowledge of the spherical figure of the earth enables us readily to determine the position of the circles of the sphere, with respect to the horizon of any place, the latitude of which is known For,

The altitude of the celestial pole at any place, is equal to the latitude of that place

Let SELNQ and HO (Fig. 5) be sections of the earth and horizon, in the plane of the meridian of the place L. LP the direction of the celestial pole, parallel to the axis SN. Then PLC = SCL, and therefore taking from each a right angle, PLO = ECL, the latitude of the place L Art 36.

40. Hence it will be easy to understand the changes of seasons over the whole earth. But it is necessary to premise that

all observers, who observe the sun at the same instant, refer it nearly to the same place in the celestial sphere. It will be shown hereafter that the greatest difference of place is 17″, and therefore we may consider the sun as appearing to describe the same great circle to all the inhabitants of the earth

41. In all places having north latitude, the portions of the northern parallels of declination above the horizon will be greater than those below the horizon, and consequently when the sun is on the northern side of the celestial equator, the days will be longer than the nights, the portions of the southern circles of declination above the horizon will be less than those below it, and therefore when the sun is on the southern side of the celestial equator, the days will be shorter than the nights The contrary will take place in southern latitudes

For all places, except at the equator and poles, the sphere (reference being had to the position of the parallels of declination, with respect to the horizon) is called an *oblique sphere*

42. At the equator the celestial poles are in the horizon, and hence the celestial equator and parallels of declination are all perpendicular to the horizon, and are bisected by it, and therefore at the equator all the heavenly bodies appear and disappear during equal times. This position of the sphere is called a *right sphere*.

43 At the terrestrial poles, the celestial poles appear in the zenith, and the celestial equator coincides with the horizon; the parallels of declination are parallels to the horizon At the north pole the southern parallels of declination are invisible, therefore the sun is there invisible during six months. This position of the sphere is called a *parallel sphere*

The circumstances mentioned in the three last articles follow from Art 39. (Fig. 6) will illustrate what has been said of an oblique sphere; (Fig 7), of a right sphere, and (Fig. 8)

of a parallel sphere. In these figures S and N represent the poles, EQ the equator, HO the horizon, and Dd Dd parallels of declination. The sphere is supposed to be viewed at right angles to the plane of the meridian, that is, all points to be transferred perpendicularly into the plane of the meridian.

44. At places having $66\frac{1}{2}°$ northern latitude, the northern parallel of declination, which is $23\frac{1}{2}°$ from the equator, will just touch the horizon; hence as the sun is in this parallel at the summer solstice, the inhabitants of those places that have $66\frac{1}{2}°$ north lat. will then observe the sun during 24 hours. The same takes place at the winter solstice for places having $66\frac{1}{2}°$ southern lat.

45. The ancients divided the globe into five principal zones. The zone extending $23\frac{1}{2}°$ on each side of the equator is called the *torrid zone*. The sun is always vertical to some place in this zone. The two zones between lat. $23\frac{1}{2}°$ and $66\frac{1}{2}°$ are called the *temperate zones*; the two zones about the poles are called the *frigid zones*. The parallel of latitude bounding the northern frigid zone is called the *arctic circle*, and that bounding the southern, the *antarctic*.

The parallel separating the torrid zone and northern temperate zone, is called the northern tropical circle; the sun, when in the beginning of Cancer, is vertical to this circle. The parallel separating the southern temperate zone from the torrid zone, is called the southern tropic: the sun when in the beginning of Capricorn is vertical to this.

The ancients also divided the globe into zones, the middle of each zone differing half an hour in the length of their longest day. From the small extent of their knowledge of the surface of the earth, they imagined that places in the same zone, which they called *climate*, differed little in temperature. If so, many parts of Siberia ought to be of the same temperature as Ireland: hence the propriety of disusing the division of the globe into climates.

CHAPTER IV.

ON REFRACTION AND TWILIGHT.

46. As connected with the earth, we may here consider its atmosphere, and how it affects the apparent places of the heavenly bodies. We know, from the science of pneumatics, that the air surrounding the earth is an elastic fluid, the density of which is nearly proportional to the compressing force, or the weight of the incumbent air. Whence it follows that the density *continually* decreases, and at a few miles high becomes very small. Now a ray of light passing out of a rarer medium into a denser, is always bent out of its course toward the perpendicular to the surface, on which the ray is incident. It follows therefore that a ray of light must be *continually* bent in its course through the atmosphere, and describe a curve, the tangent to which curve, at the surface of the earth, is the direction in which the celestial object appears. Consequently the apparent altitude is always greater than the true.

47. The refraction or deviation is greater, the greater the angle of incidence, and therefore greatest when the object is in the horizon. The horizontal refraction is about 32′. At 45° altitude, in its mean quantity it is $57\frac{1}{2}''$.

48. The refraction is affected by the variation of the quantity or weight of the superincumbent atmosphere at a given place, and also by its temperature. In computing the quantity of refraction, the height of the barometer and thermometer must be noted. The quantity of refraction at the same zenith

distance varies nearly as the height of the barometer, the temperature remaining constant The effect of a variation of temperature is to diminish the quantity of refraction about $\frac{1}{180}$ part for every increase of one degree in the height of the thermometer. Therefore, in all accurate observations of altitude or zenith distance, the height of the barometer and thermometer must be attended to.[a]

49 The refraction may be found by observing the greatest and least altitude of a circumpolar star The sum of these altitudes diminished by the sum of the refractions corresponding to each altitude, is equal to twice the altitude of the pole: from whence, (if the altitude of the pole be otherwise known), the sum of the refractions will be had, and from the law of variation of refraction, known by theory, the proper refraction to each altitude may be assigned

50 Otherwise, when the height of the pole is not known, the ingenious method of Dr Bradley may be followed, who observed the zenith distances of the sun at its greatest declinations, and the zenith distances of the pole star above and below the pole The sum of these four quantities must be 180° diminished by the sum of the four refractions, hence he obtained the sum of the four refractions, and then by theory apportioned the proper quantity of refraction to each zenith distance. In this manner he constructed his table of refractions[b]

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[a] Theory shews that, whatever be the law of change of density, the variation of refraction is as the tangent of the zenith distance, between the zenith and about 74° zenith distance At greater zenith distances we cannot apply theory to obtain the variation of refraction, because there the variation of the density of the air at different heights will sensibly affect the quantity of refraction, and the law of this variation is unknown

[b] The object of the observations in this and the preceding article is to ascertain the *coefficients* of refraction If we suppose the refraction to vary as the tangent of the zenith distance there is but one coefficient, which can be thus accurately de-

51 The ancients made no allowance for refraction, although it was in some measure known to Ptolemy, who lived in the second century. He remarks a difference in the times of rising and setting of the stars in different states of the atmosphere — This however only shews that he was acquainted with a variation of refraction, and not with the quantity of refraction itself. Alhazen, a Saracen astronomer of Spain, in the ninth century, first observed the different effects of refraction on the height of the same star above and below the pole —Tycho Brahe, in the sixteenth century, first constructed a table of refractions This was a very imperfect one.

52 As the atmosphere refracts light, it also reflects it, which is the cause of a considerable portion of the day-light we enjoy. After sun-set also the atmosphere reflects to us the light of the sun, and prevents us from being plunged into instant darkness, upon the first absence of the sun. Repeated observations shew that we enjoy some twilight, till the sun has descended 18° below the horizon From whence it has been attempted to compute the height of the atmosphere, capable of reflecting rays of the sun sufficient to reach us; but there is much uncertainty in the matter If the rays come to us after one reflection, they are reflected from a height of about 40 miles if after two, or three, or four, the heights will be twelve, five, and three miles The computation requires the assistance of

termined Let the zenith distances of the sun be S, S'; and of the star Z, Z', then by Bradley's method we have $180° = S + S' + Z + Z' + A\,.\,(\tan. S + \tan. S' + \tan Z + \tan Z')$, if the refraction be represented by $A.\tan.$ zen. dist. in general, hence A can be determined exactly See Delambre *Abregé d'Astronomie*, p 130. Ed

The investigation of the law of variation of refraction from theory, is much too difficult to find a place in an elementary book Reference may be had to Simpson's Mathematical Dissertations, Vince's Astronomy, chap. 7, p 76, Laplace's Mecanique celeste, tom IV p. 267, &c, Trans. R Irish Academy, vol. XII.

the theory of terrestrial refractions (See Professor Vince's Astronomy, Art. 206)

53 The duration of twilight depends upon the latitude of the place and declination of the sun The sun's depression being 18° at the end of twilight, we have the three sides of a spherical triangle given to find an angle, viz the sun's zenith distance (108°), the polar distance, and the complement of latitude, to find the hour angle from noon. At and near the equator, the twilight is always short, the parallels of declination being nearly at right angles to the horizon. At the poles the twilight lasts for several months, at the north pole from 22nd September to 12th November, and from 25th January to 20th March. When the difference between[a] the declination and complement of latitude *of the same name* is less than 18°, the twilight lasts all night

54 Refraction is the cause of the oval figures which the sun and moon exhibit, when near the horizon The upper limb is less refracted than the lower, by nearly five minutes, or $\frac{1}{6}$ of the whole diameter, while the diameter parallel to the horizon remains the same The rays from objects in the horizon pass through a *greater* space of a *denser* atmosphere than those in the zenith, hence they must appear less bright. According to Bougier, who made many experiments on light, they are 1300 times fainter, whence it is not surprising that we can look upon the sun in the horizon without injuring the sight.

55 Another striking phenomenon respecting the sun and moon in the horizon, must not be entirely passed over, although rather belonging to the science of optics, viz. their great apparent magnitudes The cause of this undoubtedly is the wrong judgment we form of their distances then, compared with their

[a] Or when the sun's polar distance exceeds the latitude by a quantity less than 18°.—Ed

distances when their altitudes are greater. In estimating their distances when in the horizon, we are led to judge them greater than when considerably elevated, because of the variety of intervening objects which furnish ideas. The apparent diameters being nearly the same in both cases, we are apt to judge that object largest, the distance of which we conceive greatest. This explanation is a very old one, being given by Alhazen in the ninth century. Roger Bacon, Kepler, Des Cartes, and others also, were of the same opinion.

CHAPTER V.

MICROMETERS—DIAMETERS AND DISTANCES OF THE SUN, MOON, AND PLANETS—SPOTS ON THE SUN AND PLANETS—ROTATION OF THE SUN AND PLANETS—MAGNITUDES OF THE SUN, MOON, AND PLANETS.

56. Having attained to the knowledge of the magnitude and figure of the earth, we are enabled to extend our inquiries to the magnitudes and distances of the sun, moon, and planets. The present improved state of astronomical instruments furnishes means of making observations, by which we can obtain, with considerable precision, the magnitudes of the sun, moon, and planets, and ascertain the vastness of the distances of some of them, relatively to the diameter of the earth. We can ascertain the angle two remote places on the surface of the earth subtend to a spectator at the sun, moon, or planets, and from thence deduce the angle the disc of the earth, when seen from any of these bodies, subtends. This angle can be obtained with the same accuracy as we can measure the apparent diameter of the disc of a planet. The method requires not the assistance of any theory of the arrangement of the celestial bodies, and therefore enables us to use the magnificent truths it furnishes, in establishing the true planetary system. The fixed stars appear, as was observed, precisely in the same position with respect to each other, in whatever part of the earth we are; but the planets vary their position with respect to the neighbouring fixed stars, the angular distance of a planet from a neighbouring fixed star

appearing greater in one place than in another It is from the difference of these angular distances that we obtain the angle which we should see the two places subtend, could we remove ourselves to the planet to make the observation

57 Let us proceed to consider this method more particularly, but first it may be proper to make a few remarks respecting the method of measuring small angles on the concave surface, and on the precision with which they can be measured.

The diameters of the sun, moon, and planets, that is, the angles they subtend, can be measured with much accuracy, by measuring the diameters of their images, formed by the object glass of the telescope. The image is measured by means of two parallel wires placed in the focus of the object glass. One of these wires is capable of being moved parallel to itself, so that the wires may be readily opened to touch the opposite sides of the image of a planet's disc, and the interval of the wires furnishes at once the apparent diameter of the planet, the scale being previously settled by ascertaining the opening of the wires corresponding to a given angle. This is one of the simplest kinds of micrometers in its simplest state; there are others which it is unnecessary to mention here The above is sufficient to give an idea of the method of measuring small angles Small angles can be measured with much more accuracy than large angles. In measuring large angles the whole telescope is moveable. In micrometer measures, only the small apparatus of the wires is moveable, which can be executed with much greater nicety and exactness than the aggregate parts of a large instrument. The parts of the micrometer have much greater stability than the parts of an instrument for measuring large angles. Small angles may be measured, by good instruments, with certainty, to less than 1″. The difference of declinations of two stars, having nearly the same declination, is also readily measured by moving the telescope, and turning the system of wires, so that one of the

stars moves on the fixed wire, and then moving the other wire till the other star moves along it. This may be readily done, even if the stars differ considerably in right ascension, but are so near in declination, that they are both successively seen to pass through the telescope while it remains fixed.

If the stars differ considerably in right ascension, the quantity of refraction at each observation may be changed, on account of the variation of the barometer and thermometer, and must be allowed for; but when they are near together they are both equally affected by refraction, and therefore no allowance is necessary, which is a considerable advantage.

58. To find the angle two distant places, in the same terrestrial meridian, subtend at a planet. Let H and S (Fig. 9) be two places, P a planet in the celestial meridian of these places, HF′ and SF the directions in which the same fixed star, also in the meridian at the same time, is seen at the two places. The star made use of is supposed to be very nearly in the same parallel of declination as the planet, that is, not differing in declination more than a few minutes. Produce HP to meet SF in B; then because HF′ and SF are parallel (Art. 31) HBS = BHF′; therefore HPS (= HBS + PSB) = F′HP + PSF = the sum of the apparent distances of the planet and star (the place of the planet being supposed to be between the parallels). These distances can be observed, as was said, with great accuracy, by means of a micrometer. We have thus the principal thing necessary to enable us to advance by a most important step, viz. to obtain the angle the disc of the earth subtends, as seen from a planet.[a]

[a] This angle is obtained in the following manner.

Draw the tangents PO and PO′, (Fig. 10), and the OPO′ is the angle the earth's disc subtends at the planet. Draw CHV and CSZ, C being the centre of the earth. Produce PH and PS to meet OC and O′C in D and E, and join P, C.

59. The Cape of Good Hope is nearly in the same meridian with many places in Europe, having observatories for astronomical purposes, and therefore a comparison of the observations made there, with those made in Europe, furnishes us with a means of practising this method. By a comparison of the observations of De La Caille, made at the Cape of Good Hope, with those made at Greenwich, Paris, Bologna, Stockholm, and Upsal, the angles the earth's disc subtends at Mars and at the Moon, have been obtained with very considerable precision. Comparisons of observations will also furnish the same for the sun and other planets. But it will be seen hereafter, that knowing the angle the earth's disc subtends at any one planet, we can readily find it for the sun or any other planet.

Now for the sun and planets the angle HPS is very small, and even for the moon not considerable, and therefore the distance PC is great, compared with OC. Hence we may consider OC, CD, CE as proportional to the angles OPC, CPH and CPS, and therefore OPC : CPH + CPS (= HPS) :: OC : CD + CE. But as the angles D and E are very nearly right angles, CD is the sine of the angle DHC (= PHV), and CE is the sine of CSE (= PSZ) to rad. OC. Hence OC : CD + CE :: Rad : sin. VHP + sin. PSZ and $OPO' = 2\ OPC = 2\ HPS \times \frac{\text{Rad.}}{\text{sin. VHP} + \text{sin. PSZ}}$. Thus to obtain the angle OPO' it is necessary to know the angles VHP and PSZ, or the zenith distances of the planet at the two places. But it is not necessary that these angles should be observed with much precision, since it is easy to see that an error of even a few minutes, in the quantities of these angles, will make no sensible error in the quantity $\frac{\text{Rad.}}{\text{sin. VHP} + \text{sin. PSZ}}$. The above is on the suppositions, 1st, that the star and planet are on the meridian together; 2nd, that the two places are on the same terrestrial meridian. If the *star and planet* are not on the meridian together, yet their difference of declinations being observed, it is the same as if there had been a star on the meridian with the planet. If the *two places* are not under the same meridian, an allowance must be made for the planet's motions in the interval between its passages over the two meridians, and we obtain the difference of declinations that would have been observed at two places under the same meridian.

60 The method that has been described, yields only to one other method in point of accuracy; viz to that furnished by the transit of Venus over the sun's disc, which will be particularized hereafter The above is fully sufficient for the purposes for which it is given here, which purposes are to enable us to compare the magnitudes of the sun and planets with that of the earth, and to shew the vast distances of some of them relatively to the diameter of the earth.

The diameter of the earth when nearest to and seen from

The Sun is	17″	Juno, Vesta	is	9″
Mercury -	28″	Jupiter	-	4″
Venus -	62″	Saturn	-	2″
Mars -	42″	Georgium Sidus		1″
Ceres, Pallas -	9″	The Moon		2° 2′

A planet therefore appearing to us as small as the earth appears to the inhabitants of Saturn and the Georgium Sidus, would not have been observed except by the assistance of the telescope

61 The Sun, Jupiter, Saturn, and Georgium Sidus always appear with discs nearly circular

The Moon, Mercury, Venus, and Mars exhibit variable discs, they however are always portions of circles Their diameters may be measured with micrometers, and are found to be, when greatest, as follow·

The Sun -	1920″	Jupiter	-	40″
Mercury -	11″	Saturn	-	18″
Venus -	57″	Georgium Sidus		4″
Mars -	26″	The Moon		1920″

The new planets, according to the most careful trials of Dr Herschel, appear to subtend only a small part of a second.

62 Hence we can compare the real diameters of these bodies with the diameter of the earth For,—

diameter of planet : diameter of earth :: angle planet subtends at the earth : angle earth subtends from planet.

Whence calling the diameter of the earth unity, or 8000 miles in round numbers, the diameter of

		Diam. of ⊕		Miles.
The Sun	is	111	or	888000 nearly.
Mercury	-	0,4	-	3200
Venus	-	0,9	-	7200
Mars	-	0,8	-	6400
Jupiter	-	11	-	88000
Saturn	-	10	-	80000
Georgium Sidus		4	-	32000
The Moon	-	0,25	-	2000

The largest of the new planets is supposed by Dr. Herschel not to exceed 200 miles in diameter.

63. The above method of obtaining the proportion of the diameter of a planet to that of the earth, admits of being repeated at pleasure, not being affected by the variableness of the planet's distance, and therefore a mean of many results being taken, great accuracy can be attained to.[a]

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[a] Knowing the angle the earth's disc subtends at the sun or a planet, we can ascertain the distance, because the angle in seconds subtended by the earth : 206 265 (the seconds in arc equal radius) :: diameter of the earth : distance of the planet from the earth. But a small error in the angle subtended by the earth, will occasion a considerable error in the distance, and therefore this method of ascertaining the distance is not given, as affording much precision; but it serves sufficiently for showing the vast distances of the sun and planets from the earth, which is all that is necessary for our purpose here. If the angle subtended at the sun by the earth be 17″, the sun's distance from the earth is $\frac{206265}{17} = 12133$ diameters of the earth, or 96 millions of miles *nearly*.

In like manner taking 4″, 2″, and 1″ for the angles subtended by the earth's disc at Jupiter, Saturn, and the Georgium Sidus, the distances of these planets from the earth will be 51566, 103132, 206265 diameters of the earth respectively. In this manner the mean distance of the moon from the earth is found to be about 60 *semidiameters* of the earth.

64 Having deduced the real magnitudes of the apparent circular discs, the next step is to shew that the sun and planets are spherical bodies With respect to the sun we are assisted by the consideration of its spots By the help of telescopes we often observe, on the bright surface of the sun, dark spots of various and irregular forms These appear to move on the surface from east to west, and after arriving at the western edge disappear, and after a time again re-appear on the eastern edge The times of appearance and disappearance are nearly equal, each being 13½ days nearly The deduction to be made from these circumstances is, that the spots are on the surface of the sun, for they cannot be bodies revolving about him, for then they would not appear on his surface, and disappear during equal times The sun then must revolve on an axis carrying these spots with him, or these spots must move on his surface with such a motion as will account for the phenomena The latter hypothesis is much more complicated than the former, for each spot separately must have such a motion given to it, as will solve the phenomena of its appearance and disappearance. The spots are not permanent, but are observed to increase and decrease, and at last cease to exist; yet till their entire disappearance their apparent motions on the surface of the sun continue the same, which makes it still more improbable that the motion is in the spots themselves

65. Concluding then that the sun revolves on an axis, we immediately deduce that it is a spherical body, for no revolving body but a sphere will always appear, at a distance, a circular disc The motions of the spots shew that the sun revolves on an axis inclined to the ecliptic at an angle of $82°\frac{1}{2}$, and that the time of revolution is $25^{d}\ 10^{h}$ The process of computation is too long[a] to insert here; it is sufficient to observe that calcula-

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[a] See Vince's Astronomy, vol. i. art. 395, &c.

tions from the motions of different spots give the *same* result, so that it cannot be doubted that the sun's rotation is the true cause of the appearances we observe in the motions of the spots

66. Few spots have been observed farther from the solar equator than 30 degrees. Not very unfrequently there are spots in the sun so large that they may be seen by the naked eye, when the sky is covered with a thin haziness. A spot observed in April, 1779, by Dr. Herschel, measured 1′ 8″ in diameter, and was therefore above 30000 miles in diameter, because a spot of the same diameter as the earth would only subtend an angle of 17″ (Art. 60.)

67 Various theories have been formed to explain the solar spots Astronomers generally agree that the sun is an opaque body covered by a luminous fluid, and that changes in this fluid occasion the appearance of spots. Many disputes have taken place on this subject little worth attending to, as all the hypotheses hitherto offered seem to rest upon slight foundations.

68. As the spots are occasionally seen by the naked eye, it is readily conceived they may be easily seen by the help of the most indifferent telescopes accordingly after the invention of that instrument they soon became objects of much notice. The first discovery of them is contended for by Galileo, Scheiner, and Harriot. Harriot observed them in England in December 1610, which was about the same time when Galileo mentions that he had observed them. It was not long after they were first discovered, that the inclination of the solar axis and time of revolution were ascertained.

69. By the apparent motion of spots on the discs, as well as by other arguments to be mentioned hereafter, we know that the planets Venus, Mars, Jupiter, and Saturn, are spherical bodies, each revolving on an axis.

Venus revolves in	$23^h\ 30^m$	Jupiter - -	$9^h\ 52^m$
Mars - - -	24 40	Saturn - -	10 16

The rotation of Saturn was ascertained from observation by Dr Herschel. That of Venus by M. Schroeter, a celebrated German astronomer

70 No appearances have been discovered in the other planets sufficient to determine their rotation, but it is highly probable from analogy that they revolve on axes. But we have otherwise sufficient proof of their spherical form, for if they were circular discs or hemispheres, it is highly improbable that, their motions among the fixed stars being so irregular as seen from the earth, they would always keep the same face turned toward it, for the motions being observed to be sometimes direct, and sometimes retrograde, the planet, unless it be a spherical body, must, to preserve the same circular appearance, have contrary motions about the same axis

71 The rotations of the sun and planets are all in the same direction

72 The sun and planets being spherical bodies, their magnitudes will be to that of the earth as the cubes of their diameters to the cube of the diameter of the earth, whence calling the magnitude of the earth unity, the magnitude of

The Sun is -	1367631	Jupiter - -	1281
Mercury -	0,064	Saturn - -	995
Venus - -	0,72	Georgium Sidus	80
Mars - -	0,5	The Moon -	$\frac{1}{49}$

73 The ancients had such very inadequate notions of the magnitudes and distances of the sun and planets, that the earth was considered, by them, a body of as much importance as any other in the universe Pythagoras, as may be collected from Pliny, considered the sun only three times more distant than the moon, and the moon thirteen times less distant than it is, hence according to him the sun was distant only by seven diameters of the earth instead of 12000, and so the diameter of the sun would be only $\frac{1}{15}$ of the diameter of the earth. Aristarchus, in the

third century before Christ, investigated the distance of the sun, and found it to be only 1200 diameters of the earth. Kepler, about two centuries ago, considered it nearly five times less distant than it is.

74. A spectator observing a planet not in his zenith, refers it to a place among the fixed stars, different from that to which a spectator, at the centre of the earth, would refer it. The place seen from the centre of the earth is called its true place: the arc of the great circle intercepted between these imaginary points is called the *diurnal parallax.*

75. The diurnal parallax is equal to the angle subtended at the planet by the place of the spectator and centre of the earth. For, to a spectator at H, (Fig. 10), a fixed star in the direction HV is in the zenith, and the distance of the planet from this star is VHP, but at the centre the distance is VCP, and the difference of these is the angle HPC. The diurnal parallax is greatest when the planet *appears* in the horizon; for the greatest angle that can be formed by two lines, one drawn from the planet to the centre of the earth, and the other to the surface, is when the latter is a tangent. The parallax of a planet, when in the horizon, is called the *horizontal parallax,* and is equal to the angle the semi-diameter of the earth subtends at the planet.

76. The diurnal parallax depresses an object; a planet, at rising, appears to the eastward of its true place, and at setting, to the westward, whence the term diurnal parallax. By observing the distance of a planet, at rising and setting, from a neighbouring fixed star, the angle that the earth's disc subtends at the planet may be observed, and that by one observer; but this method is not so convenient as the preceding. Observations near the horizon are uncertain: and the planet's motion in the interval of the observations requires to be most accurately known. Several other circumstances also render this method inferior to the above.

CHAPTER VI.

THE ROTATION OF THE EARTH—MOTION OF THE EARTH ABOUT THE SUN—GREAT DISTANCES OF THE FIXED STARS—PRECESSION OF THE EQUINOXES

77. HAVING acquired a knowledge of the vast distances of the sun and planets, and of their magnitudes, we are led to consider whether the diurnal motion we observe in these bodies be a real or only an apparent motion. Real and apparent motions are not at first readily distinguished from each other. The motions of a person in a ship, carriage, &c. daily afford instances that vision alone is not sufficient to distinguish between true and apparent motion. Either experience or judgment is necessary to distinguish between them.

Diurnal motion.—That the heavenly bodies really move, and, by so doing, cause the apparent *diurnal* motion, we can have no experience, nor can we readily perceive the motion of our earth, as we, in that respect, are in the same circumstances as a person in the cabin of a ship in motion. We could not easily understand whether the whole motion was in the ship, or in a bird, (the only visible external object), flying at a distance. But examining the reasons for each, we distinguish which motion is most probable, that of the earth round its axis or of all the celestial bodies in the space of $23^h\ 56^m$. Either the celestial bodies revolve in the space of $23^h\ 56^m$ in great or small parallel circles, according to their apparent distance from the celestial poles, or the cause of that apparent diurnal motion is a real mo-

tion of the earth about an axis in a direction from west to east. That the latter supposition will explain the diurnal phenomena is so evident, that it is hardly necessary to dwell upon it By the rotation of the earth about an axis, the horizon of each spectator has a motion, and will revolve in the celestial sphere instead of the sphere with its circles, so that the parts of the celestial sphere will be successively uncovered and become visible, as they would do by a motion of the imaginary sphere itself, carrying the bodies situate in it

78 The only argument against this motion is, that the spectator appears at rest and the celestial bodies appear to move. But as experience every day points out to us motions only apparent, nothing can be concluded from the apparent rest of the spectator. The arguments from analogy in favour of the rotation of the earth are very strong The Sun, Venus, Mars, Jupiter, and Saturn, all spherical bodies like the earth, (of which, three are vastly greater than the earth), revolve about their axes.

79 Also against the diurnal motions of the celestial bodies about the earth, are the vast distances and magnitudes of the sun and planets. The immense motions to be given to each of these bodies at different and variable distances from the earth, and apparently unconnected with each other and with the earth, to produce their apparent diurnal motions, would require a very complicated celestial mechanism. To suppose the sun above a million times larger than the earth, to revolve about the earth in 24 hours, instead of the earth revolving about an axis in that time, is contrary to that rule of philosophy by which effects are deduced from the simplest causes.

80 Also we know that when a body moves in the circumference of a circle, there is requisite a force tending to the centre to keep it continually in that circle. Now we can assign no force acting upon the sun and planets, to make them describe

the diurnal circles. No bodies are situate in the different centres of those circles, by the continual attraction of which they might be continually impelled from the tangent to the circumference?*

81. We conclude, then, that the diurnal motions of the celestial bodies are only apparent, and that these appearances are produced by the motion of the earth about an axis parallel to the apparent celestial axis; although every appearance may be explained by supposing the eye in the centre of a revolving sphere, in the concave surface of which the heavenly bodies are situate.

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* Although the arguments for the rotation of the earth are so satisfactory, that no doubt whatever can remain, yet it is interesting to consider whether the matter cannot be subjected to a direct experiment. It will readily appear that a body let fall from a considerable height will, if the earth revolves from west to east, fall to the eastward of the vertical line. Let C (Fig. 11) be the centre of the earth, T the place from which the body is let fall, TB the vertical line in direction of the centre. When the body reaches the earth let tb be the position of the vertical line, in consequence of the earth's motion. Take Bf = Tt and f will be the place of the body; because the body, leaving the top of the vertical with a motion equal to the motion of the top, is, at the end of its fall, as far from the first position of the vertical as the top of the vertical itself is from its first position. But Bb is less than Tt and therefore than Bf, in the proportion of CB to CT, consequently f is to the eastward of b. This is on the supposition that the place is at the equator, and it may suffice for an illustration. An accurate investigation cannot conveniently be inserted here, but may be found in Simpson's Mathematical Dissertations, and Laplace's Mecanique celeste, tom. iv. On account of the small height BT at which we can make the experiment, bf must be very small, and the utmost nicety is required. In this age, however, of accurate experiment, it has been attempted, and it is said with success. It has been tried at Bologna from the height of 257 English feet, also at Viviers and at Hamburgh; at Hamburgh the height was 250 feet, and the deviation found to be 0,35 inches to the east, and 0,13 inches to the south. Computation, not taking into the account the air's resistance, gives 0,34 inches to the east, and no perceptible deviation to the south.

Note by the Editor.—If h denote the height of the tower, and λ the latitude, the deviation to the east varies as $h^{\frac{3}{2}} \cos. \lambda$, and the deviation to the south as $h^2 . \sin. 2\lambda$.

82. The rotation of the earth has been established, beyond all controversy, since the time of Galileo, but the notion is a very old one; it is expressly mentioned by Cicero as the opinion of Hicetas, who lived about 400 years before the commencement of our æra. The words of Cicero are, "Hicetas Syracusius, ut "ait Theophrastus, cœlum, solem, lunam, stellas, supera deni-"que omnia stare censet, neque præter terram rem ullam in "mundo moveri: quæ cum circum axem se summâ celeritate "convertat et torqueat, eadem effici omnia, quasi stante terrâ "cœlum moveretur." *Acad. Quæst. Lib.* 2.[a]

83. *Annual motion.*—The apparent annual motion of the sun is explained, by supposing that either the sun moves round the earth or the earth round the sun, in a path or orbit nearly circular. For the sun, as has been stated, appears in the course of a year to describe, on the concave surface of the heavens, a great circle called the ecliptic. Observation shews that its apparent diameter does not vary much, its greatest being $= 32' 34''$ and least $31' 29''$, consequently the variation of distance, compared with the whole distance, is but small. Observations likewise shew that its apparent motion in the ecliptic or change of longitude is not equable, yet its difference from equable motion is not great. The motion for any given interval of time, if it moved equably, is found by dividing its whole motion in a year by the number of given intervals in a year. Thus it moves 360° in about 365 days, therefore in an hour the motion is $2' 28''$ nearly. This is called *the mean motion* in an hour. Its greatest hourly motion is $2' 33''$ and its least $2' 23''$. Whence in a year the sun moves in an orbit nearly circular, and with a motion nearly equable, about the earth, or the earth moves in an orbit nearly circular,

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[a] Reperi apud Ciceronem primum Hicetam sensisse terram moveri. Inde igitur occasionem nactus, cœpi et ego de terræ mobilitate cogitare. *Copernicus in suâ Præf. ad Paulum III.* Ed.

with a motion nearly equable, about the sun. That the latter motion takes place is established by a variety of reasons

84 It will be proved that the planets move about the sun in orbits nearly circular, in different periodic times and at different distances. Also that all the planets receive their light from the sun, a body vastly greater than them all in magnitude, some of which are of much greater magnitude than the earth. Again there is a certain relation between the periodic times of the planets and their distances from the sun, as will hereafter appear. Now considering the earth as a planet revolving round the sun, its distance and periodic time obey the law of the rest of the planets which circumstance affording such an harmony between the motions of all those bodies, receiving their light and apparently their heat, the source of animal and vegetable life, must at once persuade us to acknowledge the annual motion of the earth, rather than that of the sun although all the principal phenomena of the planetary motions may be explained, by supposing them to revolve in orbits nearly circular round the sun, while the sun and planets are together carried with an annual motion round the earth.

85. But the most satisfactory proof is one that we cannot introduce with its full effect here, it requiring some preliminary principles of physical astronomy This proof is from the knowledge of that universal attendant of matter, the principle of attraction or gravity. The sun, earth, and planets mutually attract each other, in proportion to their quantities of matter or their masses It follows, from the laws of motion, that they must come together, or each of them revolve in an orbit round a fixed point, the common centre of gravity of all the bodies Now we shall see hereafter that the *mass* of the sun, as well as its *magnitude*, is vastly greater than all the planets together, so much greater, that the common centre of gravity lies within the body of the sun, and the sun, in fact, will move about this point,

but in a path so small, compared with the orbits of the planets, that it may be said to be at rest, and the planets said to revolve about the sun, they revolving about a point so near his centre.

Another argument, derived from the velocity of light, will be mentioned hereafter.

86 But it is necessary to shew how this annual motion will explain the changes of the seasons, or rather how the annual motion of the earth will explain the apparent motion of the sun in a great circle inclined to the equator; for from this, as we have seen, are explained the changes of seasons.

The annual motion of the earth in an orbit, the plane of which passes through the sun, is independent of its motion round the axis. That a globe may have two motions independent of each other, one a progressive motion equally affecting each particle, and the other a rotatory motion about an axis, is easily shewn from mechanical principles. As the progressive motion affects each particle equally, it cannot affect the rotation of the globe about its axis, and therefore this axis will, while the globe has a progressive motion, remain parallel to itself. Supposing then the earth to have two such motions, it is clear that the axis cannot be perpendicular to the plane of the progressive motion, for otherwise the sun would always appear in the celestial equator. But if the polar axis be inclined to the plane of the earth's orbit constantly at an angle of 66° 32′, a spectator any where on the earth will see the sun, in the course of a year, apparently describe a great circle on the surface of the celestial sphere, inclined to the equator at an angle of 23° 28′. For the plane of the orbit constantly making the same angle with the terrestrial equator, it will intersect the surface in a great circle, inclined to the equator at an angle of 23° 28′, and therefore an eye at the centre of the earth will refer the place of the sun always seen in the plane of the orbit, to a great circle in the celestial sphere, which circle it will evidently appear to describe

in the course of a year to an eye at the centre. But it was before shewn, that, from the vast distance of the sun compared with the diameter of the earth, all spectators refer the sun nearly to the same place on the concave surface, whence we conclude, that by the motion of the earth about the sun in an orbit, to which the equator is inclined at a constant angle of 23° 28′, the sun, seen from any part of the earth, will appear to describe, in the space of a year, the great circle called the ecliptic.

87 The effects also of this inclination and parallelism of the axis, will readily appear, by considering that a hemisphere (or rather somewhat more) of the earth, the base of which is perpendicular to the line joining the centres of the sun and earth, is illuminated by the sun. The positions of the poles and parallels of latitude with respect to this hemisphere, will easily shew the variation of the length of the days and of seasons.

Let HVTP (Fig 12, 1) represent the path or orbit of the earth about the sun S; let also AB represent the axis of the earth, B being the north and A the south pole Conceive this axis in a plane at right angles to the orbit, and that this plane always continues parallel to itself, while the centre of the earth moves about the sun, the axis will then, it is evident, also move parallel to itself. Let AHB be the position of the axis when this plane passes through the sun, and the angle SHB = 90° + 23° 28′ When the centre H has moved a right angle about the sun to V, this imaginary plane being parallel to its former position, SV must be at right angles to it, that is, to every line in it, therefore SVB is a right angle When the centre comes to T in SH produced, the plane again passes through the sun, and because TB and HB are parallel, STB = 90° − 23° 28′, and is then least When it comes to P opposite to V, again SPB is a right angle H will represent the place of the earth at the winter solstice, V at the vernal equinox, T at the summer solstice, and P at the autumnal equinox. For, Fig.

12, 2 will represent the earth at H with its enlightened and dark hemispheres, seen at right angles, to the plane of the meridian passing through the sun The angle SHB is greater than in any other position, and the north pole B will be in the dark hemisphere farthest removed from the circle of light and darkness The parallel of lat Lm is the arctic circle, and will just touch the circle[a] of light and darkness All places on the north side of the equator, will have a greater portion of their parallels of latitude in the dark than in the enlightened hemisphere, and therefore the days will be shorter than the nights The equator is equally divided, and the parallels on the southern side have a greater portion in the enlightened than in the dark hemisphere rs will be the parallel to which the sun is vertical, and will represent the southern tropical circle, because rHe = LHB = 23° 28′.

V will be the place of the earth at the vernal equinox, for, Fig. 12, 3 will represent the earth at V with its enlightened and dark hemispheres, viewed at right angles to the plane of the meridian passing through the sun. The circle of light and darkness will pass through the poles and equally divide the parallels of latitude, therefore all places will have equal day and night, and the sun will be vertical to the equator.

T will be the place of the earth at the summer solstice, for, Fig. 12, 4 will represent the earth at T, with its enlightened and dark hemispheres viewed as before, and the same may be remarked with respect to the northern and southern hemispheres, as was observed with respect to the southern and northern when the earth was at H. Fig. 12, 3 may also represent the earth when at P, with its enlightened and dark hemispheres

88. An objection to the motion of the earth must be consi-

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[a] The circle called the circle of light and darkness, is the circle, which is the boundary, between the dark and enlightened hemispheres.

dered here, which at first sight may appear to have some weight No change is observed in the relative position of the fixed stars, in consequence of that motion The angular distances of the fixed stars, observed at different seasons of the year, always remain the same, even when observed with the most exquisite instruments But, supposing the motion of the earth in an orbit, nearly circular, round the sun, the observer in one situation is nearer some stars by 24000 diameters of the earth, (vid. note, page 45), than in another, and consequently the angular distances of those stars ought to appear greater.[a]

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[a] Let TDE (Fig 13) represent the orbit of the earth, T and E the places of the earth at the solstices, when the axes Pp, P′ p′ of the earth are in a plane which passes through the sun, and is perpendicular to the plane of the orbit. Let F be a fixed star in this perpendicular plane. When the earth is at T the observed distance of the star from the celestial pole is FTP, when at E it is FEP′. Produce pP to meet FE in R then the angle F = TRE — FTR = FEP′ — FTP. But these angles are constantly the same, not having any perceptible difference, and therefore the angle subtended by the diameter of the earth's orbit, at a star situate in the solstitial colure, is imperceptible Dr Bradley took much pains to ascertain the angle F in the case of γ Draconis, a star of the second magnitude, situate nearly in the plane above mentioned or in the solstitial colure, about 15° from the pole of the ecliptic. This star passing the meridian near his zenith, admitted of being observed by a zenith sector, an instrument particularly adapted for observing with great precision near the zenith, where also no error can occur from the uncertainty of refraction He found the angle F imperceptible by his observations. My own observations, and those of Mr Pond, the present Astronomer Royal, agree also *as to this star*, in shewing that the angle F is imperceptible. Let us suppose the angle F = 2″, draw the perpendicular EK, then 2″ : 206265″ (the seconds in an arch = radius) :: sin 2″ : rad :: EK : FE. But EK : TE :: sin. ETK : radius For γ Draconis the angle ETK = 75° nearly, hence EK = ,97.TE, and therefore FE = $\frac{206265}{2}$ × ,97.TE = 100000.TE nearly. If the earth therefore move about the sun, the distance of γ Draconis must be at least 200000 times greater than the distance of the sun from the earth, or above two thousand million diameters of the earth.

The greatest angle the diameter of the earth's orbit subtends at any fixed star, which is called the parallax of the star, has been, till lately, thought impercepti-

89. The distance of the fixed stars, proved by the motion of the earth, is indeed wonderful, yet there is nothing contrary to our reason or experience in admitting it. Why should we limit the bounds of the universe by the limits of our senses? We see enough in every department of nature to deter us from rejecting any hypothesis, merely because it extends our ideas of the creation and divine Creator.

The best telescopes do not magnify the fixed stars, so as to submit their diameters to measurement, but it is well ascertained that the apparent diameter of the brightest of them is less than 1″. Now being self shining bodies, and not subject, except in a few instances, to any apparent alteration, we may conclude them to be bodies of the nature of our sun. But that the diameter of the sun may appear less than a second, it must be removed 1900 times farther from us than at present; which is an argument in favour of the vast distance of the fixed stars. It must however be confessed, that this argument from analogy is much too weak to be in any degree decisive, and our positive

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ble. M. Piazzi, from his observations made at Palermo, suspected a parallax of a few seconds in several stars (Vid. Con. des temps, 1808, p. 432). Particular attention has been paid by myself to this subject, and my observations made with the circle, 8 feet in diameter, belonging to the Observatory of Trinity College, Dublin, appeared to point out a parallax in several stars. The agreement of results obtained by different sets of observations, seemed to leave no doubt on this head. However, observations made elsewhere do not confirm my results. An opportunity will offer further on of again mentioning this question.

Note by the Editor—The celebrated Hooke was the first person that asserted the existence of annual parallax. His object was that it should serve as an *experimentum crucis* to determine between the Tychonic and Copernican systems. Hooke's observations, however, were too inaccurate to be at all relied on. Flamstead also asserted the existence of parallax, but did so from having confounded it with aberration. Bradley completely separated these inequalities, and denied the existence of sensible parallax for the fixed stars.

knowledge of the immense distance of the fixed stars must depend upon the certainty of our knowledge of the earth's motion, of which we have such evidence as must be considered conclusive.

90. *Precession of the equinoxes* —Although the place of the celestial pole among the fixed stars has been considered as not changed by the annual motion of the earth, yet in a longer period of time it is observed to be changed, and also the situation of the celestial equator; while the ecliptic retains the same situation among the fixed stars. Observation shews that this change of situation of the pole and equator is nearly regular. The pole of the celestial equator appears to move with a slow and nearly uniform motion, in a lesser circle, round the pole of the ecliptic, while the intersections of the equator and ecliptic move backward *on the ecliptic*, with a motion nearly uniform This motion is at the rate of about 1° in 72 years, or more accurately 50″, 2 in a year; consequently the sun returns again to the same equinoctial point before he has completed his revolution in the ecliptic; so that the equinoxes *precede* continually the complete apparent revolution of the sun in the ecliptic: and hence the term precession of the equinoxes In consequence of this apparent motion all the fixed stars increase their longitudes by 50″, 2 in a year, and also change their right ascensions and declinations. Their latitudes remain the same. The period of the revolution of the celestial equinoctial pole about the pole of the ecliptic is nearly 26000 years.

The north celestial pole therefore will be, about 13000 years hence, nearly 49° from the present polar star; and about 10000 years hence, the bright star *α* Lyræ will be within 5° of the north pole This star therefore which now in these latitudes, passes the meridian within a few degrees of the zenith, and twelve hours after is near the horizon, will then remain nearly stationary with respect to the horizon. All which will readily

appear, from considering the celestial concave surface as represented by a common celestial globe

91 This motion of the celestial pole originates from a real motion in the earth, whereby its axis, preserving the same inclination to its orbit, has a slow retrograde conical motion. The cause of this motion is shown, by physical astronomy, to arise from the attraction of the sun and moon on the excess of matter at the equatorial parts of the earth By physical astronomy we are also enabled to account for a small change in the plane of the ecliptic Observations, separated by a long interval, point out that the obliquity of the ecliptic is diminishing at nearly the rate of half a second in a year, that is, the ecliptic appears approaching the equator by half a second in a year. Physical astronomy shews that this arises from a change in the plane of the earth's orbit, occasioned by the action of the planets: that this change of obliquity will never exceed a certain small limit. and that by this action of the planets, the ecliptic is progressive on the equator 14″ in a century [a]

The precession of the equinoxes is not entirely uniform, for a small inequality in the precession, and change in the obliquity of the equator to the ecliptic, depending on the position of the moon's *nodes* (the intersections of its path and the ecliptic) were discovered by Dr Bradley, and are confirmed by physical astronomy. The poles of the equator describe round their mean places a small ellipse, not differing much from a circle about 18″ in diameter, in 18 years [b]

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[a] Hence the annual precession arising from the spheroidical figure of the earth is 50″, 10 + 0″, 14 = 50″, 33 annually

[b] There is also a solar inequality of precession, depending on the place of the sun in the ecliptic, this is never greater than 1″, 1.

92. The precession of the equinoxes was first discovered by Hipparchus. As the quantity of it is so perceptible in a hundred years, a comparison of the position of the circles of the sphere, as recorded in the earliest æra of astronomy, and of their position now, has been used to assist chronology.

CHAPTER VII.

ON THE MOTIONS OF THE PRIMARY PLANETS—THE SOLAR OR COPERNICAN SYSTEM—THE PTOLEMAIC SYSTEM

93 Having stated some of the principal arguments for the motion of the earth, in an orbit nearly circular about the sun, let us now consider the planets in general. Astronomy has added much indeed to our knowledge of the creation, by enabling us to ascertain that the planets are vast bodies, revolving round the sun in orbits nearly circular, some at greater and others at less distances than the earth, that some of these bodies are smaller and others much larger than the earth. and that, according to a high degree of probability, they are bodies of the same nature as that on which we live.

94. The principal planets are always observed to be nearly in the ecliptic, the annual path of the sun on the concave surface, and for the present let us consider them as seen in the ecliptic.

The most striking circumstance in the planetary motions is the apparent irregularity of those motions, the planets one while appearing to move in the same direction among the fixed stars as the sun and moon, at another in opposite directions, and sometimes appearing nearly stationary. These irregularities are only apparent, and arise from a combination of the motion of the earth and motion of the planet; the observer, not being conscious of his own motion, attributing the whole motion to the planet

95. The planets really move, according to the order of the

signs, in orbits nearly circular, and with motions nearly uniform, round the sun in the centre, at different distances, and in different periodic times The periodic time is greater or less, according as the distance is greater or less Upon the hypothesis that the planets thus move, we can ascertain, by help of observation, their distances from the sun, and thence compute, for *any time*, the place of a planet, which is *always* found to agree nearly with observation.

96. First, for those planets which are limited in their elongation from the sun The *elongation* of a planet from the sun is the angle subtended at the earth by the sun and planet. These planets are nearer the sun than the earth is, and therefore called inferior planets. The greatest elongation of the inferior planet Mercury from the sun is about 28°, and the greatest elongation of Venus is about 47°

The interval of time between two successive inferior conjunctions with the sun can be observed A planet is said to be in *inferior conjunction*, when it comes between the sun and the earth. In *superior conjunction*, when the sun is between the earth and planet In inferior conjunction, the planet being nearest to the earth, appears largest, and may be observed with a good telescope, even a very short time before the conjunction. For our purpose here, it is not necessary that the time of conjunction should be observed with great accuracy. Let T represent the time between two successive inferior conjunctions. Then, to a spectator in the sun, in the time T, the inferior planet (moving with a greater angular velocity) will appear to have gained four right angles, or 360° on the earth, and the planet and earth being supposed to move with uniform velocities about the sun, the angle gained (the angle at the sun between the earth and planet, reckoning according to the order of the signs), will increase uniformly.

97. Let TEL represent the orbit of the earth, DNPGO that

of an inferior planet, each being supposed circular, S the sun in the centre, and P the place of the planet when the earth is at E. Then in the triangle SEP (Fig. 14) we obtain the angle SEP the elongation by observation,[a] and the angle PSE by computation, for it is the angle the planet has gained on the earth since the preceding inferior conjunction. Therefore, this angle PSE : 360° :: time from inferior conjunction : T. The two angles SEP and PSE being known, the angle SPE is known, and hence SP *relatively* to SE, for sin. SPE : sin SEP :: SE : SP. Having thus obtained the distance of the planet from the sun, we can, *at any time*, by help of the time T and the time of the preceding inferior conjunction, compute the angular distance of the planet from the earth, as seen from the sun, and thence, by help of the planet's and earth's distance from the sun, compute the planet's elongation from the sun. Thus the planet being at O, and the earth at E, we can compute the angle ESO, and having the sides SE and SO, we can, by trigonometry, compute the angle SEO the elongation of the planet from the sun. This being compared with the observed angle, we always find them nearly agreeing, and thereby is shown that the motions of the inferior planets, Mercury and Venus, are explained by those planets moving in orbits nearly

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[a] The ancients observed the places of the fixed stars and planets with respect to the sun, by the assistance of the moon or planet Venus. In the day time they very frequently could observe the situation of the moon with respect to the sun Venus also being occasionally visible to the naked eye in the day time, they used that star for the same purpose. Now we can, owing to the convenience of our instruments, without the intervention of a third object, obtain the angular distance of a planet from the sun, by observing the declinations of each, and the difference of their right ascensions. By which we have, in the triangle formed by the distances of each from the pole of the equator and from each other, two sides and the included angle to find the third side, the angular distance of the planet from the sun.

circular about the sun in the centre. As the computed place always agrees with the observed place, it necessarily follows that the retrograde, stationary appearances, and direct motions, of these planets, are explained, by assigning these circular motions to them

98. It is easy to demonstrate the retrograde and stationary appearances

To do this more clearly, it will be necessary to consider the effect of the motion of the spectator arising from the motion of the earth, in changing the apparent place of a distant body. The spectator, not being conscious of his own motion, attributes the motion to the body, and conceives himself at rest Let S be the sun, (Fig. 15) ET the space described by the earth in a small portion of time which therefore may be considered as rectilinear. The motion is from E toward T Let V be a planet, supposed at rest, any where on the same side of the line of the direction of the earth's motion as the sun Draw EP parallel to TV, then while the earth moves through ET, the planet supposed at rest will appear to a spectator, unconscious of his own motion, to have moved by the angle VEP, which motion is *direct*, being the same way as the apparent motion of the sun And because the earth appears at rest with respect to the fixed stars, the planet will appear to have moved forward among the fixed stars by the angle VEP = EVT = the motion of the earth, as seen from the planet supposed at rest. Thus the planet being on the same side of the line of direction of the earth's motion as the sun, will appear, as far as the earth's motion only is concerned, to move direct. Let M be a planet any where on the opposite side of the line of direction, then the planet will appear to move retrograde by the angle MER And therefore, as far as the motion of the earth only is concerned, a planet, when the line of direction of the earth's motion is between the sun and planet, will appear retrograde

99. To return to the apparent motion of the inferior planets. Let the earth be at E, (Fig. 14), and draw two tangents GE and ED. Then when the planet is at D or G, it is at its greatest elongation from the sun S. It is clear that the planet being in the inferior part of its orbit between D and G, relatively to the earth, and the earth being supposed at rest, the planet will appear to move from left to right, that is, retrograde: and in the upper part of the orbit from right to left, that is, direct. But the earth not being at rest, we are to consider the effect of its motion. In the case of an inferior planet, the planet and the sun are always on the same side of the line of direction of the earth's motion, and therefore the effect of the earth's motion is always to give an apparent direct motion to the planet, (Art. 98). Hence in the upper part of the orbit between the greatest elongations, the planet's motion will appear direct, both on account of the earth's motion and its own motion. In the inferior part of the orbit the planet's motion will only be direct, between the greatest elongation and the points where the retrograde motion from the planet's motion becomes equal to the direct motion from the earth's motion. At these points the planet appears stationary: and between these points, through inferior conjunction, it appears retrograde.

100. Next, for the superior planets, or those planets which are farther from the sun than the earth is. The interval of time between two succeeding oppositions of a superior planet to the sun can be observed. A superior planet is in *opposition*, when the earth is between the sun and planet. It is known when a superior planet is in opposition, by observing when it is in the part of the zodiac opposite to the place of the sun. Let T represent the time between two successive oppositions, then viewing the planet from the sun, the earth will appear to have gained an entire revolution, or 360° on the planet, in the time T; and the earth and planet being supposed to move with *uniform* an-

gular velocities about the sun, the angle gained by the earth will increase uniformly.

101 Let TEL (Fig 16) represent the orbit of the earth, CDOG that of a superior planet, N the place of the planet when the earth is at E Then, in the triangle SNE, we have the angle SEN by observation, and the angle NSE by computation For NSE is the angle at the sun which the earth has gained on the planet since the preceding opposition This angle 360° · time since opposition · T The two angles NSE and SEN being known, the angle SNE is known, and therefore SN relatively to SE For sin SNE : sin SEN :: SE : SN. Having thus obtained the distance of a superior planet from the sun, we can, *at any time*, by help of the time T, and time of preceding opposition, compute the angular distance of the earth from the planet, as seen from the sun, and thence, by help of the earth's distance and planet's distance from the sun, we can compute the planet's elongation from the sun. Thus the planet being at R and the earth at E, we compute the angle RSE, and knowing the sides ES and SR, we can (by plane trig.) compute the angle RES, the elongation of the planet from the sun. This being compared with the observed angle, we *always* find them nearly agreeing, and thereby is shewn that the motions of the superior planets are explained, by those planets moving in orbits nearly circular about the sun As the computed place nearly agrees with the observed place, it necessarily follows that the retrograde and direct motions, and the stations, of these planets are explained, by assigning to them these circular motions.

102 And it is easy to demonstrate these appearances It is clear that the planet being in any part of its orbit, and the earth being supposed at rest at any point E, the planet will appear to move from west to east, or direct But the earth not being at rest, we are to consider the effect of its motion. The earth being at E, draw the tangent DEG, then if the planet is

in the upper part of the orbit DCG, it is on the same side of the line of direction of the earth's motion as the sun, and therefore the effect of the earth's motion is to give an apparent direct motion to the planet The earth being at E, and the planet at D or G, the planet is said to be in quadrature; consequently from quadrature to conjunction, and from conjunction to quadrature, the planet appears to move direct, both on account of its own motion and the motion of the earth If the planet is in the lower part of the orbit DOG, the effect of the earth's motion is to give an apparent retrograde motion to the planet, consequently from quadrature to opposition, and from opposition to quadrature, the planet moves direct or retrograde according as the effect of the planet's motion exceeds, or is less than, the effect of the earth's motion. Between quadrature and opposition their effects become equal, and the planet appears stationary, and afterward through opposition to the next station retrograde.

103. The apparently irregular motions of the planets among the fixed stars, must strike the most cursory observer, and it would not at first be expected that these motions could be explained by so simple an arrangement of the bodies But it is not enough to establish the true arrangement and true motions of the bodies, that the general appearances are explained It is necessary that the most minute circumstances of their apparent motions can be shewn to arise from that arrangement We have supposed above that the orbits are accurately circular, that the planes of these orbits and that of the earth coincide, and that the angular motions were uniform; but if the planes of the orbits coincided, if the orbits were accurately circular, and were uniformly described, the planets would always appear in the ecliptic, and would always be found exactly in the places which the computation on the circular hypothesis points out; but none of these things take place exactly The deviation however can be explained, by

shewing, that the planes of the orbits of the planets are inclined to the plane of the earth's orbit at small angles, and that the orbits are not circles, but only nearly circles, being ellipses, not differing much from circles, as will be shewn farther on. Every phenomenon, even the most minute, can be deduced from such an arrangement; no doubt therefore would remain of the motions of the planets, in such orbits, round the sun, even had we not the evidence derived from physical astronomy

Another arrangement, known by the name of the Ptolemaic system, will explain the general appearances of the planetary motions, will shew when they are direct, stationary, and retrograde, and will enable us to compute nearly their apparent places, but when applied to the more minute circumstances of their motions, it totally fails.

104 The periodic times of the inferior planets can be deduced nearly, from observing the time between two conjunctions, their orbits being supposed circular

Let T = the time between two successive inferior or superior conjunctions

E = periodic time of the earth

P = periodic time of the planet

Then considering the planet's angular motion as uniform, P : E :: 4 right angles : angle described by planet about the sun in time of earth's revolution = 4 right angles + angle gained by planet on earth in time of earth's revolution

But the angles gained are as the times of gaining them; therefore 4 right angles : 4 right angles + angle gained by planet on earth in time of earth's revolution :: T : T + E.

Hence[a] P : E :: T : T + E, therefore $P = \frac{T \times E}{T+E}$, conse-

[a] Otherwise thus —The angle described by the planet in the unit of time is $\frac{360}{P}$, and that by the earth $\frac{360}{E}$, hence their separation in this time is $\frac{360}{P} - \frac{360}{E}$,

quently knowing the time between two inferior conjunctions, which can be readily observed, we obtain the periodic times of the planets Mercury and Venus.

The interval between the inferior conjunctions of Mercury is 115 days, therefore its periodic time $= \frac{115 \times 365}{115 + 365} = 87$ days

The interval for Venus is 584 days, and consequently its periodic time $= \frac{584 \times 365}{584 + 365} = 224$ days

105 The periodic times also of the superior planets can be obtained, from observing the time between two successive oppositions

Let T, E, and P represent as before Then P : E :: 4 right angles : angle described by planet in time of earth's revolution = 4 right angles — angle gained by earth on planet in time of earth's rev. Also 4 right angles : 4 right angles — angle gained by earth in time E :: T : T — E, hence P : E :: T : T — E, therefore $P = \frac{T \times E}{T - E}$.

The interval between two oppositions of the Georgium Sidus is $369\frac{3}{4}$ days; hence the periodic time of the Georgium Sidus $= \frac{369,75 \times 365,25}{4,5} = 82 \times 365\frac{1}{4} = 82$ years For Saturn, the interval is 378 days, and consequently the periodic time of Saturn $= \frac{378 \times 365\frac{1}{4}}{378 - 365\frac{1}{4}} = 29\frac{1}{2} \times 365\frac{1}{4} = 29\frac{1}{2}$ years In like manner the periodic times of the other superior planets may be nearly determined.

106. The inclinations of the planes of the orbits of all the

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but since they separate by 360 in the time T their separation in the unit of time is also $\frac{360}{T}$: equating these quantities we have $\frac{1}{P} - \frac{1}{E} = \frac{1}{T}$ whence $P = \frac{TE}{T+E}$. For the superior planets the equation is $\frac{1}{E} - \frac{1}{P} = \frac{1}{T}$, whence $P = \frac{TE}{T-E}$ —Ed.

planets, except Pallas, to the plane of the earth's orbit are small. The method of ascertaining the inclinations will be afterward shewn. The points, in which a planet's orbit intersects the plane of the earth's orbit, are called *nodes*. The node through which the planet passes from the southern to the northern side of the ecliptic, is called the *ascending node*, and the other the *descending node*.

When an inferior planet is near one of its nodes at inferior conjunction, it appears a dark spot on the sun's surface, and thereby is shewn that the inferior planets receive their light from the sun. When Venus is in superior conjunction, at a considerable distance from its node, it may be seen, by help of a telescope, to exhibit an entire circular disc. Indeed all the different appearances of the inferior planets, as seen through a telescope, are consistent with their being opaque bodies, illuminated by and moving about the sun in orbits nearly circular. Near inferior conjunction they appear crescents, exhibiting the same appearance as the moon a few days old. At the greatest elongation they appear like the moon when halved, and between the greatest elongation and superior conjunction they appear gibbous, or like the moon between being halved and full.

107. These appearances are easily explained.—The planet being a spherical body, the hemisphere turned toward the sun is illuminated. A small part only of this hemisphere is turned toward the earth, when the planet is near inferior conjunction. Half the enlightened hemisphere is turned toward the earth, when the planet is at its greatest elongation. More than half, when the planet is between its greatest elongation and superior conjunction.

For, generally, both with respect to inferior and superior planets, the greatest breadth of the part of the illumined hemisphere turned toward the earth, is proportional to the exterior angle at the planet, formed by lines drawn from the planet to

the sun and earth. Let PS (Fig 17) be in the direction of the sun, PE in that of the earth, IPHLO the section of the planet in the plane of the earth's orbit. Draw HO perpendicular to EP, and HIO is the greatest breadth of the hemisphere turned toward the earth; IL being perpendicular to SP, IHL is the greatest breadth of the illuminated hemisphere; and HI common to each, is the greatest breadth of the illuminated part seen from the earth. The measure of this is the angle IPH = IPS + SPH = HPG + SPH = SPG the exterior angle at the planet. Now near inferior conjunction the exterior angle is less than a right angle, at the greatest elongation it is a right angle; and afterwards greater than a right angle. Therefore the breadth of the illuminated part is respectively less than a quadrant, equal to a quadrant, and greater than a quadrant

108 It is easy to see that as the planets appear flat discs on the concave surface, so their illumined parts will be projected on the flat surface, and the greatest breadth will be projected into its versed sine, as in Fig. 18 1, 18. 2, 18 3, where IH is projected into its versed sine AB Because the projection of a circle, inclined to a surface, by right lines perpendicular to that surface, is an ellipse, the inner termination PS of the enlightened part appears elliptical, and the enlightened surface surface of planet : : AB : AC : : versed sine of exterior angle : diameter

109 With respect to the superior planets, the exterior angle of the planet is least when the planet is in quadrature. For when the exterior is least the interior is greatest. Now it is evident that SGE, (Fig. 16) when GE is a tangent to the orbit of the earth, is greater than when E is at any other point, and therefore the planet being in quadrature, the exterior angle is least SGE for every superior planet is acute, and the exterior angle obtuse, and consequently its versed sine is greater than radius Whence more than half the disc of a superior planet is

always seen, and it appears most gibbous in quadrature Mars then appears gibbous about $\frac{1}{8}$ of his diameter; Jupiter only by about $\frac{1}{100}$ of his diameter, which quantity is imperceptible, even by a telescope, because Jupiter's disc then only subtends an angle of 30″ Accordingly all the superior planets, except Mars, appear always with a full face. The new planets appear so small, that it cannot be expected that they should appear in any degree gibbous

110. The brightness of a planet depends both on the quantity of illuminated surface and its distance The greater the distance is, the less the brightness, which, the illuminated surface remaining the same, decreases as the square of the distance increases, so that in computing when a planet appears brightest, both the illuminated surface and distance must be taken into the account Both circumstances concur in making a superior planet appear brightest at opposition The inferior planets are not brightest at superior conjunction, because of their greater distance; and near inferior conjunction, the illuminated part visible to us is very small. The place of greatest brightness then lies between inferior and superior conjunction.

The solution of the problem to find when Venus appears brightest, gives her elongation then about 40 degrees. The places of greatest brightness are between the places of greatest elongation and inferior conjunction This agrees very well with observation When she is near this position she occasions a strong shadow in the absence of the sun, and for a considerable time both before and after she is at this elongation, she may be readily seen in full day-light by the naked eye.

111. From inferior to superior conjunction Venus is to the westward of the sun, and therefore rises before the sun, and by the splendor of her appearance, being much noticed, is called the morning star From superior to inferior conjunction she appears to the eastward of the sun, and therefore does not set

till after the sun, and is then called the evening star. Jupiter, which approaches much nearer in splendor to Venus than any other planet, is sometimes called a morning or evening star, according as it rises before or sets after the sun, and when near opposition may be called both an evening and morning star.

112. The following TABLE exhibits at one view the principal outlines of the planetary system

	Merc ☿	Ven ♀	Earth ⊖	Mars ♂	Vesta ⚶	Juno ⚵	Ceres ⚳	Pallas ⚴	Jup ♃	Sat ♄	Geor ♅	Sun ☉
Mean distances from sun, earth's dist being 10.	4	7	10	15	24	27	28	28	52	95	192	
Periodic time	days 87	days 224	days 365	days 686	days 1335	days 1592	days 1681	days 1681	years 12	years 29½	years 83	
Diameter, earth's diam. 10	4	9	10	8					110	100	43	1128
Inclination of orbit to ecliptic	7°	3° 23′		1° 51′	7° 9′	13° [illegible]′	10° 27′	34° 39′	1° 19′	2° 30′	0° 46′	
Place of ascending node as seen from sun	15°	74°		46°	103°	171°	81°	172°	97°	111°	73°	
Diameter of sun seen from planet	80′	46′	32′	21′	13′	12′	11′	11′	6′	3′	1′	
Times of revolution on axes		h m 23 30	h m 23 56	h m 24 40					h m 9 52	h m 10 16		d h 25 10
Days from conj to con or opp to op	115	584		780	503	474	466	466	399	378	369½	
Of which time they retrograde, during days.	22	42		70	83	99	99	99	120	135	151	
Arcs which they retrograde.	12°	16°		18°	13°	12°	12°	12°	9°	6°	4°	
Velocity per second in miles.	30	23	19	15	13	12	11	11	8	6	4	
Greatest and least apparent diam	11″ 5″	57″ 10″		26″ 5″					40″ 20″	18″ 15″	4″	

The times and arcs of retrogradation are computed on the supposition that the orbits are circular.

The apparent diameters of the new planets have not been ascertained. They are too small to be measured by micrometers.

Dr Herschel thinks that if the diameter of any one of them amounted to $\frac{1}{4}$ of a second, he should have been able to have ascertained it.—Phil. Tran. Part 1, 1805.

The minuteness of these bodies has induced him to class them as distinct from the planets, under the name of Asteroids. It may be observed that an apparent diameter of $\frac{1}{4}$ of a second in opposition would give a real diameter of 222 miles.

113. Perhaps the most striking circumstance in the above table, is the great velocities with which the planets move; and this is more impressed, when we consider that of the earth on which we live, the velocity of which is 90 times greater than the velocity of sound. In contemplating these velocities, it cannot but occur to us how great a power is necessary to be continually acting, to circumflect the planets about the sun, and compel them to leave the tangential direction. A power that acts incessantly, and is able to counteract the great velocities of the planets, must excite our inquiries as to its origin and law of action.

We can ascertain that this power is constantly directed towards the sun, increases in intensity as the square of the distance from the sun decreases, and that it is the same power which is diffused through the whole planetary system, only varying in quantity as the square of the distance from the sun is varied. So far physical astronomy teaches, but the proximate cause of this power, or solar gravity, as it may be called, is unknown. We cannot trace by what agency the Supreme Being, from whom all things originate, has ordained the operations and laws of gravity to be executed.

114 By a comparison of the distances and periodic times, which are determined independently of each other, it will be seen that *the squares of the periodic* times are as the *cubes of the distances.* This relation was first found out by Kepler For a long time no necessary connexion was discovered between the periodic times and distances, till at last it was shown to be a consequence of the law of gravity above-mentioned

115. At present we know of no secondary cause that could have any influence in regulating the respective distances of the planets from the sun, yet there appears a relation between the distances, that cannot be considered as accidental This was first observed by Professor Bode of Berlin, who remarked that a planet was wanting, at the distance at which the new planets have since been discovered, to complete the relation. According to him, the distance of the planets may be expressed nearly as follows, the earth's distance from the sun being 10.

Mercury	4	=	4
Venus	$4+3\times 1$	=	7
Earth	$4+3\times 2$	=	10
Mars	$4+3\times 2^2$	=	16
New planets	$4+3\times 2^3$	=	28
Jupiter	$4+3\times 2^4$	=	52
Saturn	$4+3\times 2^5$	=	100
Georgium Sidus	$4+3\times 2^6$	=	196

Comparing these with the mean distances above given, we cannot but remark the near agreement, and can scarcely hesitate to pronounce that these mean distances were assigned according to a law, although we are entirely ignorant of the exact law and of the reason for that law.

116. Astronomy must have been considerably advanced before any attempts were made to ascertain the position of the planets with respect to the sun and to each other, and to develope their motions. It is said, however, that the Egyptians very

early conceived the motions of the planets Mercury and Venus to be about the sun, and also that the Pythagoreans considered the sun as the centre about which the planets performed their motions. But their opinions are so imperfectly expressed in the few scattered notices which are found in different authors, that little can be known with certainty about them

The distinguished astronomers of the Alexandrian school, Aristarchus, Eratosthenes, Hipparchus, and others, seem not to have attempted any theory of the planetary motions, notwithstanding they far excelled in other parts of astronomical knowledge all that had gone before And we are certain that till Ptolemy, who wrote about 140 years after the birth of Christ, published the system that goes by his name, the motions of the planets were not submitted to regular calculation

In the Ptolemaic system, the earth is supposed immoveable in the centre, about which the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn are supposed to revolve in different periods and in the order stated. All these bodies, as well as the fixed stars, were likewise supposed to be carried round the earth by the motion of the *primum mobile* in 24 hours. The latter opinion appears now so unphilosophical, that we are apt to judge by it of the rest, and despise the whole Ptolemaic system, as unworthy of consideration. However, that part of the system by which the inequalities of the planetary motions were explained is well worthy of examination, and seems in some measure entitled to the credit which it possessed for near fourteen centuries

117 The motions of the inferior planets were supposed to be as follow. Let E (Fig 19) be the earth, SS′ the path of the sun, V Venus in inferior conjunction. Venus is supposed to move uniformly in a circle which is carried uniformly round the earth Let V N be the circle in which Venus moves, while this circle is moved uniformly about the earth. The circle in

which the planet moves is called the *epicycle*, and that on which the centre of the epicycle moves is called the *deferent*. The epicycle is described in the time between two inferior conjunctions of the planet, and the deferent is described in the time of the earth's revolution about the sun. It is easy to see that such a combination of motions will represent the motion of the planet. At V the motion of the planet being towards N, and that of the epicycle towards D′, the former motion about E exceeding the latter, the planet appears retrograde when seen from E. But it will readily appear generally, that the angular distance of the planet from the sun is always rightly represented in this system, and therefore the apparent motion of the planet. When D has moved to D′, let V′ be the place of the planet. Produce ED to S′, and S′ will be the place of the sun; because the time of describing the deferent is the same as the period of the sun's motion. The angle V′D′E will answer to the angle gained by the planet on the earth in Art. 97, and Fig. 14. Hence if the radius of the deferent : radius of epicycle :: SE : SP (Fig. 14) :: distance of earth from sun : distance of planet from sun in the true system; the triangle EV′D′ will be always equiangular to the triangle ESP; and therefore as we have shown that the angle SEP rightly represents the elongation of the planet from the sun, S′EV′ will also rightly represent the elongation, and therefore this system will rightly represent the motion of the inferior planets.

118. The motions of the superior planets were supposed to be in epicycles, each described in the time between two conjunctions or oppositions; but the deferents were described in the same times as the planets revolve round the sun in the true system, that is, the epicycle of Saturn was described in 378 days, and the deferent in 29½ years. Let DD′ (Fig. 20) be the deferent of a superior planet, M the planet in opposition, the sun being at S. When the centre of the epicycle is at D′, let M′

be the place of the planet and S′ that of the sun, produce D′E to P. Then M′DE will answer to the angle gained by the earth on planet in Art. 101, and Fig. 16, but S′ES − DED′ = angle gained, because the deferent is described in the periodic time of the planet. Hence M′D′E = S′ES − PES = PES′; therefore D′M′ and S′E are parallel, and consequently M′ES′ = D′M′E. But if the radius of the epicycle : radius of the deferent :: SE : SN (Fig. 16) :: distance of earth from sun : distance of planet from sun in true system; the triangle ED′M′ will be always equiangular to SEN (Fig. 16). Hence D′M′E, and therefore S′EM′ will always shew the true angular distance of the sun from the planet, and so the motions of the superior planets will be rightly represented.

119. There are some circumstances in the Ptolemaic system that ought naturally to have led to the true system. The former determines nothing with respect to the distances of the planets from the earth; it only requires that the proportion of the radii of the deferent and epicycle be such as to represent the motion for each planet. The distances therefore are arbitrary. If we take the radius of the deferent of an inferior planet equal to the radius of the sun's orbit, we immediately have the inferior planets revolving round the sun, while the sun is carried round the earth, according to the reported system of the Egyptians. This simplification of the Ptolemaic system with respect to the inferior planets is so obvious, that we may suppose it soon occurred without any reference to the Egyptian system, and to have been the first advance toward the true system. We know it is mentioned by Martianus Capella, who appears to have lived in the fifth century, and by others long before the time of Copernicus. If we take the radius of the deferent of a superior planet equal to the planet's true distance from the sun, the radius of the epicycle for each planet will be the earth's distance from the sun. This striking circumstance might have led Copernicus

cus to simplify the system, by giving a motion to the earth, by which one circle is made to serve the purpose of several equal ones

120 Although the Ptolemaic system explains the general appearances with much simplicity, yet when it was applied to explain those appearances which arise from the inclination of he orbits to the ecliptic, from the eccentricities and the unequal motions in those orbits, the introduction of other circles beside the deferent and epicycle being necessary, the system became very complex, and much ingenuity and mathematical sagacity were shewn in adapting it to different circumstances Iad the instruments now in use then existed, a very few observations would have been sufficient to have completely overthrown all those speculations But the state of instruments and f observations was such in the time of Copernicus, after whom ie true system has justly been named, that he could use scarcely ny arguments in support of his system but what he derived om its simplicity It was only a short time before his death, 1543, at the age of 71, he ventured to propose his system to e world, in his work entitled "De Revolutionibus Orbium," ter having meditated upon it above 36 years It does not em to have made much impression till above half a century ter, when Galileo, aided by his telescope, was enabled to bring ost powerful arguments in favour of it. His observation of e gibbosity of Venus was decisive in favour of the motion of enus about the sun. Had the motion of Venus been accordr to the Ptolemaic system, it must always have appeared in a escope as a crescent.

121. The ancients observing that the planets moved faster slower according to the place of the ecliptic they were in, en in opposition, or near conjunction, named this *the first quality* The retrograde, stationary, and direct appearances y called the *second inequality* Copernicus, who conceived

that the celestial motions were necessarily performed in circles, was obliged to retain epicycles to explain the first inequality

122. Although there was nothing in the Ptolemaic system, that could properly lead to the knowledge of the actual distances of the planets from the earth, yet as the system appeared very imperfect without it, astronomers substituted an hypothesis resting on no foundation. They imagined that the convex boundary of the space, within which the epicycle of a planet performed its motion, was the concave boundary of the space belonging to the next; and as they knew, although inaccurately, the distance of the moon, they obtained from it the distance of Mercury, from the distance of Mercury that of Venus, &c. The distances obtained in this way differed extremely, as might be expected, from the truth. Till therefore the Copernican system was established, nothing whatever was known with respect to the actual distances, and consequently the magnitudes, of any of the planets But the distances of the sun and moon, although very inaccurate, were deduced from just principles.

CHAPTER VIII.

ON THE SECONDARY PLANETS AND MOON—ATMOSPHERES OF PLANETS—RINGS OF SATURN—COMETS.

123. Four small stars, only visible by the help of telescopes, always accompany Jupiter, and are continually changing their positions with respect to each other and Jupiter. They are called satellites and secondary planets. The first satellite is that which elongates itself least from Jupiter, &c. They clearly shew that Jupiter is an opaque body enlightened by the sun; for when they intervene between him and the sun, they project a shadow on his disc. They themselves are also opaque bodies illuminated by the sun; for when the planet intervenes between any of them and the sun, they are eclipsed. The phænomena prove that they revolve about their primary at different distances in orbits nearly circular, while they are carried together with their primary about the sun. Their orbits are inclined to the plane of Jupiter's orbit, as is concluded from the unequal durations of the eclipses of the same satellite. The fourth satellite is sometimes in opposition to the sun, without being eclipsed. This is owing to the inclination of its orbit and great distance from Jupiter. The third and fourth satellites disappear and re-appear on the same side of Jupiter. Only the beginnings or the endings of the eclipses of the first and second satellite are visible.

124. Let S (Fig. 21) be the sun; I Jupiter and its shadow; A and P the earth before and after the opposition of Jupiter;

sp the path of the first satellite in the shadow, A*t* a tangent to Jupiter. When the first satellite enters the shadow, the apparent distance of the satellite from the body of Jupiter is *t*A*s*, but at its emersion, the line *p*A always passes through Jupiter, and therefore the emersion is invisible; but after opposition, the earth being at P, the emersion and not the immersion will be visible. The same things take place with respect to the second satellite If *mn* be the path of the third satellite, *m*A frequently lies without the body of Jupiter, and therefore both the immersion and emersion are visible, and the phænomena are very striking, from the circumstances of the satellite disappearing and re-appearing at a distance from the body of Jupiter on the same side The same may be observed with respect to the fourth satellite Before the opposition of Jupiter to the sun, the eclipses happen on the west side of Jupiter; after opposition, on the east. If the telescope invert, the contrary takes place.

125 It has long been suspected, that the satellites of Jupiter revolve on their axes, and lately Dr Herschel has observed that each of them revolves in the time of its revolution round the primary.[a] Their motions about the primary, and their motions about their axes, are from west to east

126 Their distances in semi-diameters of Jupiter, and their periodic times are nearly as follow:

Sat	Dist	Per	Sat	Dist	Per
I	6	$1^d\ 18^h$	III	14	$7^d\ 4^h$
II	9	$3^d\ 13^h$	IV	26	$16^d\ 16^h$

They must be very magnificent objects to the inhabitants of Jupiter. The first satellite appears to them with a disc four times greater than that of our moon appears to us, and goes through all the changes of our moon in the short space of 42

[a] Phil. Trans. 1797, page 332

hours, within that period being itself eclipsed, and causing an eclipse of the sun on the surface of Jupiter

127 The order of their magnitudes is 3d, 4th, 1st, 2d, according to Dr Herschel. Their masses, that of the earth being 1000, and therefore of the moon 14, are

Sat	Mass	Sat	Mass
[a]I - -	5	III - -	27
II - -	7	IV - -	13

128 The satellites of Jupiter, at their greatest elongations, appear nearly in the direction of the equator of Jupiter, because the equator of Jupiter and the orbits of the satellites are inclined at small angles, to the plane of Jupiter's orbit The direction of Jupiter's equator is marked by the *belts* of Jupiter, which are faint shades, parallel to each other, on the body of Jupiter, and which frequently undergo such changes, that they have been supposed to be somewhat of the nature of clouds in his atmosphere; but, from some unknown cause, more permanent than our clouds.

129 Galileo discovered the four satellites of Jupiter, Jan 7, 1610. This, which might naturally have been a source of delight, was at first a subject of disappointment He supposed them to be fixed stars, and found, looking at them on the next night, that Jupiter was to the eastward of them, whence he concluded the motion of Jupiter direct, whereas, according to the Copernican system, it ought then to have been retrograde, but he soon discovered that the motion was in what he took for fixed stars, and announced his discovery to the world Harriot also appears to have discovered them about the same time that Galileo did

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[a] These masses are according to the determination of M. La Place, (Mecan. Celest tom 4, p 126) It has been thought right to mention their masses, as well as some other particulars of the satellites, although they require investigations that could not properly be stated here.

This discovery was very important in its consequences It furnished, as we shall see, a ready method of finding the longitude of places by means of the eclipses of the satellites that so frequently take place This made the eclipses be particularly attended to, which led Roemer to discover that the transmission of light is not instantaneous; and this led Bradley to account for a small apparent motion of the fixed stars, called the aberration of light, which has furnished an independent proof of the motion of the earth, as strong as that from physical considerations.

130 Saturn has seven satellites revolving about him in orbits nearly circular Of which the sixth is seen without much difficulty, and was called the Huygenian satellite, from having been discovered by Huygens The 3d, 4th, 5th, and 7th were afterward discovered. Dr. Herschel discovered the first and second.

It has long been supposed that the 7th (formerly the 5th) satellite revolved on its axis in the time of its revolution round Saturn This has been confirmed by the observations of Dr. Herschel These satellites, except the sixth, require a very good telescope to render them visible. On which account they have been much less attended to than the satellites of Jupiter. The distances from Saturn in semi-diameters of Saturn, and periodic times, are nearly as follow:

Sat		Dist		Per	Sat		Dist.		Per.
I	-	2,8	-	0d 22h	V	-	8,7	-	4d 12h
II	-	3,5	-	1d 8h	VI	-	20,3	-	15d 22h
III	-	4,8	-	1d 21h	VII	-	59,1	-	79d 7h
IV	-	6,3	-	2d 17h					

131. Dr Herschel long ago discovered two satellites to the Georgium Sidus. Their orbits are nearly perpendicular to the orbit of their primary. He has since observed four others.

The relation of the periodic times, and distances of the sa-

tellites from their primary, holds in all the secondaries of each planet respectively.

132. Next to the sun, the most interesting to us, of all the celestial bodies, is our own satellite, the moon. It apparently describes, by a motion from west to east, on the concave surface of the celestial sphere, a great circle nearly, intersecting the ecliptic at an angle of about 5°. This apparent motion is explained by a real motion round the earth, in an orbit inclined to that of the earth, at an angle of 5°. The periodic time, or time of return to the same point of the concave surface, or the same fixed star, is 27 d. 7 h. 43 m. The variation of diameter shews the variation of distance is greater than the variation of the sun's distance. The greatest diameter is $33'\frac{1}{2}$, least $29'\frac{1}{2}$, and the mean $31'\frac{1}{2}$. The moon is carried with the earth in its annual motion round the sun. This necessarily follows, if the motion of the earth be granted, and is well illustrated by the motion of the satellites of Jupiter and Saturn. The apparent motion of the moon on the celestial concave surface varies considerably from its mean quantity, and its variations seem very irregular. Its greatest hourly motion in its great circle is $33'\ 40''$, its least $27'$, mean $32'\ 56''$; so that in its mean quantity it moves over an arch equal to its apparent diameter in about an hour.

133. The intersections of its apparent path with the ecliptic, or the intersections of its orbit and the earth's orbit, called its *nodes*, are not fixed, but move backward, completing a revolution in 6798 days = 18 years 228 days. If we conceive then, a great circle inclined to the ecliptic, at an angle of 5 degrees, and a body moving in this circle at the rate of about $33'$ in an hour, while the circle itself is carried backward with a slow motion of $8''$ an hour, the path of this body on the concave surface will in some measure represent the path of the moon. The more accurate considerations of the lunar motions will be resumed hereafter. The full investigation of the motions of the moon is

one of the most intricate, and, as connected with finding the longitude at sea, one of the most useful problems in astronomy. Perhaps in no instance has modern science reaped so much credit as from the success that has followed the attempt to completely develope the lunar motions

134 The phases of the moon are particularly interesting, they prove the moon to be a spherical body illumined by the sun. When in conjunction with the sun, the moon is invisible: when, moving from the sun toward the east, it is first visible, it is called the new moon, and appears a crescent · when 90 degrees from the sun it is halved, when more distant it is gibbous, and when in opposition, it shines with a full face; approaching the sun toward the east, it becomes again gibbous, then halved, and lastly a crescent, after which it disappears, from the superior lustre of the sun, and the smallness of the illumined part which is turned toward the earth

135. The enlightened part varies nearly as the versed sine of the angle of elongation from the sun It is proved in the same manner[a] as for the planets, that the enlightened part varies as the versed sine of the exterior angle at the moon. But, this exterior angle is equal to the angle of elongation + angle subtended at the sun by the earth and moon The latter angle never amounts to 10′, and therefore is inconsiderable.

136. The time between two conjunctions or two oppositions called a *lunation*, and *synodic month*, is greater than the time of a revolution in the orbit, or the time of return to the same fixed star. Because, when the latter time is completed, the moon has to move a farther space to overtake the sun

Let S = period of sun's apparent motion about the earth.

P = period of moon's motion about the earth

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[a] Art. 107 and 108

L = period between conjunction and conjunction, or of a lunation

Then S . P . 4 right angles angle described by sun in the moon's periodic time = angle gained by the moon in the time L — P

But the angles gained by the moon are as the times of gaining them Therefore,
4 right angles . angle gained by moon in time L — P . : L : L — P. Hence

[a]S : P · L · L — P or S · L · P . L — P or S . S + L . . P : L, therefore $P = \frac{S \times L}{S + L} = \frac{365,25 \times 29,53}{391,78}$ = 27 days, 7 hours, 40 minutes nearly.

137. In 19 solar years of 365¼ days, there are 235 lunations and 1 hour Therefore, considering only the mean motion, at the end of 19 years, the full moons fall again upon the same days of the month, and only one hour sooner. This is called the Metonic Cycle, from Meton, who published it at the Olympic Games, in the year 433 B C. This period of 19 years has been always in much estimation for its use in forming the calendar, and from that circumstance, the numbers of this cycle have been called the golden numbers

138. The cause of the appearance of the whole moon, observed a few days before and after the new moon, is the reflection of light from the earth. When the moon becomes consi-

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[a] Or thus :—The separation in the unit of time is $\frac{360^\circ}{P} - \frac{360^\circ}{S}$, it is also $\frac{360^\circ}{L}$; hence we have $\frac{1}{P} - \frac{1}{S} = \frac{1}{L}$ and therefore $P = \frac{S\,L}{S + L}$ The quantity L here used may be computed as follows, let the mean daily motions of the sun and moon about the earth be a and b, then in the time L the separation will be L . $(b - a) = 360^\circ$, therefore $L = \frac{360^\circ}{b - a}$ —Ed.

derably elongated from the sun, it is then out of the way of this reflection. This phænomenon affords a remarkable proof, that of two objects of the same magnitude, the brighter object appears larger.

139 One of the earliest attempts upon record to discover the distance of the sun from the earth, was from observing when the moon was exactly halved or dichotomised. At that time the angle at the moon, formed by lines drawn from the moon to the sun and earth, is exactly a right angle, therefore if the elongation of the moon from the sun be exactly observed, the distance of the sun from the earth will be had, that of the moon being known, by the solution of a right angled triangle, that is, sun's distance . moon's distance · rad cos moon's elongation. The uncertainty in observing when the moon was exactly dichotomised, rendered this method of little value to the ancients However, by the assistance of micrometers, it may be performed with considerable accuracy Vendelinus, observing at Majorca, the climate of which is well adapted to observation, determined, in 1650, the sun's distance, by this method, very considerably nearer than had been done at that time by any other method.

This method is particularly worthy of attention, being the first attempt for the solution of the important problem of finding the sun's distance It was used by Aristarchus of Samos, who observed at Alexandria, about 280 years before the commencement of the christian æra

140. Viewing the moon with a telescope, several curious phenomena offer themselves Great variety is exhibited on her disc. There are spots differing very considerably in degrees of brightness Some are almost dark Many of the dark spots must necessarily be excavations on the surface or valleys between mountains, from the circumstances of the shades of light which they exhibit There is no reason to suppose that there is any large collection of water in the moon, for if there were, when

the boundary of light and darkness passes through it, it must necessarily exhibit a regular curve, which is never observed. The non-existence of large collections of water is also probable from the circumstance of no changes being observed on her surface, such as would be produced by vapours or clouds; for, although, as will be remarked, the atmosphere of the moon is comparatively of small extent, yet it is probable that an atmosphere does exist

141. That there are lunar mountains is strikingly apparent, by a variety of bright detached spots almost always to be seen on the dark part, near the separation of light and darkness.

These are tops of eminences enlightened by the sun, while their lower parts are in darkness. But sometimes light spots have been seen at such a distance from the bright part, that they could not arise from the light of the sun Dr. Herschel has particularly noticed such at two or three different times. These he supposes are volcanoes He measured the diameter of one, and found it $=3''$, which answers to four miles on the surface of the moon

142. The heights of lunar mountains may be ascertained by measuring with a micrometer the distance between the top of the mountain, at the instant it first becomes illuminated, and the circle of light and darkness This measurement is to be made in a direction perpendicular to the line, joining the extremities of the horns

Let ADB (Fig. 22) be the circle of light and darkness, T the top of a mountain just illumined by the ray DT coming in a direction perpendicular to the plane of the circle ADB, and being a tangent to the surface at D Let S be at the surface, or the bottom of the mountain, and C the centre of the moon; then (by Euclid, 3 B. 36) $TS(TS+2CS)=DT^2$ or, TS being very small compared with CS, $TS\times 2\,CS=DT^2$, or $TS=\frac{DT^2}{2CS}$ We cannot measure DT directly, because we observe

only the projection of DT on the plane of the circle of vision AOB Now DT is perpendicular to the plane of the circle ADB, and therefore makes an angle with the plane of the circle of vision = the complement of the spherical angle OAB Therefore DT observed = DT × sine OAB = DT × sine[a] of the angle of elongation of the moon from the sun Hence DT = $\frac{\text{DT observed}}{\text{sin. elong}}$ and consequently $\text{TS} = \frac{(\text{DT observed})^2}{2\text{CS} \times \text{sin.}^2 \text{ elong}}$

Old writers on astronomy, when mentioning this problem have not considered that the projection of DT was only measured, and not DT itself, as has been remarked by Dr. Herschel. Their methods therefore only held when the moon was elongated 90° from the sun[b]

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[a] Art. 135

[b] Ricciolus mentions that, on the fourth day after new moon, he observed the top of the hill, called St Catherine's, to be illuminated, and that it was distant from the confines of the lucid part, about a sixteenth of the moon's diameter. Hence computing according to his method, that is, supposing DT itself $\frac{1}{16}$ part of the moon's diameter, and calling the moon's diameter unity,

$\text{TS} = \frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$ part of the moon's diameter, and as the moon's diameter = 2000 miles nearly, $\text{TS} = \frac{2000}{256} = 8$ miles nearly, the height of St Catherine's according to Ricciolus but on the fourth day after new moon, the moon could not be further elongated from the sun than 48° Therefore TS could not be less than $\frac{8 \text{ miles}}{\sin^2 48°}$ $= \frac{8}{(,74)^2} = 14\frac{2}{3}$ miles nearly But later astronomers are not inclined to allow so great an elevation to any of the lunar mountains. Dr Herschel investigated the heights of a great many, and he thinks that, a few excepted, they generally do not exceed half a mile But there seems to be little doubt that there are mountains on the surface of the moon, which much exceed those on the surface of our earth, taking into consideration the relative magnitudes of the moon and earth. M Schroeter determined the height of one, called Leibnitz, to be 25,000 feet, whereas the height of Chimborazo is not 20,000 feet so that, taking into consideration the relative magnitudes of the earth and moon, this lunar mountain will be five times higher than any of the terrestrial mountains.

143. It is not the least remarkable circumstance of the moon, that it always exhibits nearly the same face to us. We always observe nearly the same spots, and that they are always nearly in the same position with respect to the edge of the moon. Therefore as we are certain of the motion of the moon round the earth, we conclude that the moon must revolve on an axis nearly perpendicular to the plane of her orbit, in the same time that she moves round the earth, viz. in 27½ days. This must necessarily take place in order that the same face may be continually turned toward the earth during a whole revolution in her orbit. The motion of the moon in her orbit is not equable, therefore if the rotation on her axis be equable, there must be parts in her eastern and western edges, which are only occasionally seen. These changes, called her *libration in longitude*, are found to be such as agree with an equable motion of rotation. There are parts about her poles only occasionally visible. This, called her *libration in latitude*, arises from her axis being constantly inclined to the plane of her orbit, in an angle of 86°. A *diurnal libration* also takes place; at rising, a part of the western edge is seen, that is invisible at setting, and the contrary takes place with respect to the eastern edge. This is occasioned by the change of place in the spectator, occasioned by the earth's rotation.

144. A few remarks may be here made concerning the rising and setting of the moon, at different seasons, and of some other circumstances of moon-light.

The rising and setting of the moon is most interesting at and near full moon. At full moon, it is in or near that part of the ecliptic, opposite to the sun. Hence at full moon, at midsummer, it is in or near the most southern part of the ecliptic, and consequently appears but for a short time above the horizon; and so there is little moon-light in summer, when it would be useless. In mid-winter, at full, it is near or in the northernmost

part of the ecliptic, and therefore remains long above the horizon, and the quantity of moon-light is then greatest when it is most wanted, and this is the more striking, the nearer the place is to the north pole. There, at mid-winter, the moon does not set for fifteen days together, namely, from the first to the last quarter

145 The moon, by its motion from west to east, rises later every day, but the retardations of rising are very unequal In northern latitudes, when the moon is near the vernal intersection of the ecliptic and equator, or the beginning of Aries, the retardation of rising is least, and when near the beginning of Libra, greatest. This will appear by considering that when Aries is rising, the part of the ecliptic below the horizon makes the least angle with the horizon, and when Libra is rising, the greatest [a]

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[a] To explain this more fully, let H′CH (Fig 23) represent a portion of the horizon, CL a portion of the ecliptic when the beginning of Aries is at C, and CMN a portion of the equator Suppose the moon to rise at C on one night, then after a revolution of the concave surface, the circles will come again into the same position with respect to the horizon, but the moon will have advanced, suppose to L (in this illustration we consider the moon as moving in the ecliptic). Let HL be a parallel to the equator, then on the second night the moon will rise nearly at H, and therefore HM and LN being secondaries to the equator, MN + $23^h\ 56^m$ or CN — CM + $23^h\ 56^m$ will be the interval elapsed between two successive risings. If CL′ be a portion of the ecliptic when the moon in Libra is rising, and L′ the place of the moon on the second night, then H′ will be nearly the place of its rising, and the interval will be M′N + $23^h\ 56^m$ or CN + CM′ + $23^h\ 56^m$ It will readily appear that CM′ = CM, because LN = L′N nearly Hence the retardation, when the moon rises in Libra, is greater than the retardation when the moon is in Aries, by 2CM reduced to time It is easy to see that these are the two extremes of retardation. The angle LCN = obliquity of the ecliptic, and H′CM′ = HCM = compl of Lat.

Hence by spherical trigonometry,

sine LN (HM) = sine ob ecl × sine CL

tan CN = cos ob ecl. × tan. CL

sine CM = tan. lat × tan HM.

146 The variation of the retardation of rising, according as the moon is in or near different parts of the ecliptic, being understood, the explanation of the *harvest moon* is very easy.

At the full moon nearest the autumnal equinox, the moon is observed to rise nearly at sunset, for several nights together. This moon, for its uses in lengthening the day, at a time when a continuance of light is most desirable to assist the husbandman in securing the fruits of his agricultural labours, is called the *harvest moon*

The moon, at full, being near the part of the ecliptic, opposite to the sun, and at the autumnal equinox the sun being in Libra, consequently the moon must be then near Aries, when, from what has been stated, the retardation of her rising only amounts to a few minutes; and as the moon at full always rises at sunset, the cause of the whole phenomenon is apparent In

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Now CL in its mean quantity is about 12°, and therefore for lat. 53°, 23′, we shall find by actual computation,

$$\left.\begin{array}{l} CN = 11^\circ\ 2' \\ CM = \ \ 6\ \ 21 \end{array}\right\} \begin{array}{l} \text{Hence } CN - CM = 4^\circ\ 38' \text{ or in time} = 18^m\ 32^s \\ \text{and } CN + CM = 17^\circ\ 26' \text{ or in time } 1^h\ 9^m\ 41^s \end{array}$$

Hence the interval between the rising of the moon on two different nights, when in Aries $= 23^h\ 56^m + 18\frac{1}{2}^m = 24^h\ 14\frac{1}{2}^m$ nearly, and the retardation is only $14\frac{1}{2}^m$. When the moon rises in Libra, the interval is $23^h\ 56^m + 1^h\ 9\frac{3}{4}^m = 25^h\ 5\frac{3}{4}^m$, and the retardation is $1^h\ 5\frac{3}{4}^m$ *

This difference is still greater, the nearer we approach the Arctic circle, and there the retardation of rising, when in Aries, becomes smaller, for then HCN (the comp of lat) approaches to equality with LCN, the obliquity of the ecliptic, and therefore the points H and L approach each other, and consequently MN becomes smaller. At the Arctic circle itself, the ecliptic coincides with the horizon, when Aries is rising, and MN vanishes, and therefore the interval between two successive risings is only $23^h\ 56^m$ So that there the moon actually rises four minutes sooner

* This effect will be increased from the inclination of the moon's orbit to the ecliptic, when the ascending node is between Capricorn and Cancer, and decreased, when between Cancer and Capricorn.

places near the Arctic circle the phænomenon is still more striking, and there it is of greater use, where the changes of seasons are much more rapid.

ON THE ATMOSPHERES OF THE PLANETS AND MOON.

147 In tracing analogies between the planet on which we live and the other planets, we naturally enquire respecting their atmospheres The atmosphere which surrounds the earth has such various and important uses, that we can hardly suppose the planets destitute of an element, of which we know not whether the simplicity of construction, or the complicated advantages of it, are most to be admired.

We can ascertain that Venus, Mars, and Jupiter are surrounded by transparent fluids, which reflect and transmit light, and are therefore, according to much probability, of the same nature as our atmosphere

The spots and belts of Jupiter are not exactly stationary on his disc, but are observed to undergo changes and small motions similar to what would be observed, from a distance, of the clouds of our atmosphere; whence they are supposed to be clouds in his atmosphere; from some cause unknown to us, more permanent than any of the clouds of the earth. From observing the revolutions of some spots at different times, Dr. Herschel has discovered a difference very similar to what would arise, did monsoons take place in the atmosphere of Jupiter, as they do in that of the earth.

Appearances in Mars strongly indicate the existence of an atmosphere. A small star, hid by Mars, was observed to become very faint before its appulse to the body of Mars.

But the existence of an atmosphere about Venus, as dense, or probably denser than that of the earth, seems to be put beyond all doubt, by the observations of M. Schroeter. He, for

a series of years, observed Venus with great attention with reflecting telescopes of his own and of Dr. Herschel's making, and also with achromatic telescopes.

The[a] results of his observations are, that Venus revolves on an axis, in $23^h\ 21^m$, has mountains like the earth, and enjoys a twilight. He, in several favourable circumstances, when Venus was seen a thin crescent, measured the extension of light beyond the semicircle of the crescent, and found it to be such, that the observed zone of Venus, illuminated by twilight, must have been at least four degrees in breadth. Now for the twilight to be seen by us through the atmosphere of Venus and our own, extending through such an arch, makes it very probable that the inhabitants of Venus enjoy a longer twilight than those of the earth, and that her atmosphere is denser.

148. The existence of an atmosphere in some of the planets being ascertained, we are led to make inquiry with respect to the satellites. We can have little hopes of being able to ascertain the point, except in our own satellite, the moon.

Many astronomers formerly denied the existence of an atmosphere at the moon; principally, from observing no variation of appearance on the surface, like what would take place, did clouds exist as with us: and also, from observing no change in the light of the fixed stars on the approach of the dark edge of the moon. The circumstance of there being no clouds, proves either that there is no atmosphere similar to that of our earth, or that there are no waters on its surface to be converted into vapour: and that of the lustre of the stars not being changed, proves that there can be no dense atmosphere. But astronomers now seem agreed that an atmosphere does surround the moon, although of small density when compared to that of our

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[a] Phil. Trans. 1795.

earth. M. Schroeter has observed a small twilight in the moon, such as would arise from an atmosphere capable of reflecting the rays at the height of about a mile.

Had the moon an atmosphere of considerable density, it would readily be discovered by the durations of the occultations of the fixed stars. The duration of an occultation would be sensibly less than it ought to be, according to the diameter of the moon. The light of the star passing by the moon would be refracted by the lunar atmosphere, and the star rendered visible when actually behind the moon; in the same manner as the refraction by the earth's atmosphere enables us to see the celestial objects for some minutes after they have actually sunk below our horizon, or before they have risen above it. Now the duration is certainly never lessened eight seconds of time, which proves that the horizontal refraction at the moon must be less[a] than 2″, which therefore shews that if a lunar atmosphere exists, it must be 1000 times rarer than the atmosphere at the surface of the earth, because the horizontal refraction by the earth's atmosphere is nearly 2000″. With such a rare atmosphere, the lunar inhabitants must be deprived of many of the advantages we enjoy, from the existence of our own. Indeed the loss of one advantage, that of twilight, is, on account of the length of their day, not of much consequence, and from the apparent irregularities of the lunar surface so much light may be reflected, that the assistance of the atmosphere to make day-light, may not be so necessary as with us.

149 The existence of a solar atmosphere is also made probable by some circumstances, or an atmosphere external to the luminous atmosphere, which, according to the opinion of many

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[a] For the duration being lessened by 8″, the beginning of the occultation would be retarded 4″ of time, during which the moon moves over 2″ of space.

This seems to be caused by a double horizontal refraction, if so the lunar atmosphere must be 2000 times rarer than the terrestrial. ED.

astronomers, covers the opaque body of the sun Bouguer, by some curious experiments, on the light of different points of the disc of the sun, found the light from the centre stronger than from the borders, which seems to shew that the light from the borders is rendered weaker by an atmosphere

OF THE RINGS OF SATURN

150 Soon after the invention of telescopes, a remarkable appearance was observed about Saturn. After a considerable interval of time, Huygens having much improved them, discovered, by careful observations, a phænomenon unique, as far as we know, in the solar system. He found that Saturn is encompassed with a broad thin ring, inclined by a constant angle of about 30° to the plane of Saturn's orbit; and therefore at nearly the same angle to our ecliptic, and so always appearing to us obliquely. When its edge is turned toward us, it is invisible, on account of its thinness not reflecting light enough to be visible, except in the very best telescopes When the plane of the ring passes between the earth and sun, it is also invisible, because its enlightened part is turned from us, and when it passes through the sun, it is also invisible, the edge being only illuminated; so that it may have, in the same year, two disappearances and re-appearances. This takes place when Saturn is near the nodes of the ring.

151. The ring is a very beautiful object, seen in a good telescope when in its most open state. It then appears elliptical, its breadth being about half its length. Through the space between the ring and the body, fixed stars have sometimes been seen The surface of the ring appears more brilliant than that of Saturn himself

152 Among the numerous discoveries of Dr. Herschel, those he has made with respect to Saturn and his ring are not the least. He has ascertained that the ring, which heretofore

had generally been supposed single, consists of two exactly in the same plane, and that these both revolve on their axes[a] in the same time as Saturn, and in the plane of Saturn's equator. He also saw the ring when it had disappeared to other observers, either from the reflection of the edge, or from the dark side enlightened by the reflection of Saturn, as we see the whole moon near new moon. He observes that the ring is very thin, compared with its width, its thickness being only about 1000 miles

The outside diameter of the larger ring is -	200000
Its width - - - - -	6700
Distance between rings - - -	2800
Outside diameter of smaller ring - -	180000
Its width - - - - -	19000

At the mean distance of Saturn, the apparent diameter of the larger ring is $47''\frac{1}{2}$

153 Dr Herschel tells us, he suspects two rings to the Georgium Sidus, perpendicular to each other, but at present can only hint at so curious a circumstance.

ON COMETS

154. Comets are luminous bodies, occasionally appearing, and generally in the part of the heavens, not far from the sun. They are not so bright as the planets, but have somewhat of a nebulous appearance They do not appear long together, some are seen only for a few days, and those that appear longest, only for a few months It is probable that they receive their light from the sun, although this cannot be exactly proved. In the direction of their motion about the sun, they differ from the planets, some being direct, and others retrograde Their paths, with respect to the ecliptic, are also very different · some move in a

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[a] It ought to be noticed that this is doubted by Harding and Schroeter. See Conn des Temp 1808, p 420

direction nearly perpendicular to it. But the most striking phænomenon, and what makes them objects of attention to all mankind, is the tail of light which they often exhibit When approaching the sun, a nebulous tail of light is seen to issue from them in a direction opposite to the sun this, after having increased, again decreases till it disappears. The stars are visible through it

155 Very many comets have been recorded in history, the motions of at least one hundred have been computed. It may be sufficient to observe here, that they move about the sun in eccentric ellipses, the sun being in one of the foci The other considerations of their orbits and motions, are deferred till after the account of the discoveries of Kepler Little would have been known on this subject but for the discoveries of Kepler and Newton; and although the discoveries of Kepler might by analogy have led to a knowledge of the motion of comets, yet nothing of consequence was done till Newton himself illustrated the subject.

156. The appearance of one comet has been several times recorded in history, viz the comet of 1680. The period of this comet is 575 years It exhibited at Paris a tail 62° long, and at Constantinople one of 90°. When nearest the sun it was only $\frac{1}{6}$ part of the diameter of the sun distant from his surface; when farthest, its distance exceeded 138 times the distance of the sun from the earth.

157. When the theory of the motion of comets was understood, Dr. Halley examined the comets that had been previously recorded in history, and been observed by astronomers In general, he found the circumstances so vaguely delivered, or the observations so inaccurately made, that he was able to determine with much probability the identity of only one comet. He supposed also that the comets observed in 1532 and 1661 were the same, and, that therefore it might be expected again

in 1789; but it did not appear. However Dr. Halley was very doubtful of their identity, on account of the imperfection of the observations of Apian in 1532. Farther notice will be taken of this, when we mention more particularly the return of comets. The comet, which Dr. Halley predicted with a degree of confidence, returned in 1759. It had been previously observed with accuracy, in 1682 and 1607, and had also been noticed in 1531, 1456, and 1305. Its return was anxiously looked for by astronomers, and some curious circumstances attending it will be afterwards noticed. With what satisfaction it was received by the scientific part of mankind may easily be conceived, and how strikingly contrasted with the reception of the same comet in 1456, when all Europe beheld it with fear and amazement. The Turks were then engaged in the successful war, in which they destroyed the Greek empire, and Christians in general thought their destruction portended by its appearance. We may be nearly certain that this comet will re-appear again in 1834.

158. With respect to the tails of comets, little satisfactory can be offered, in recording the various opinions on this subject. According to Sir Isaac Newton, they arise from a thin vapour, sent out from the comet, by the heat of the sun, and supported in the solar atmosphere.

This hypothesis has been controverted by several authors, and very ably by Dr. Hamilton, late Bishop of Ossory.

Dr. Hamilton supposes the tails of comets, the *aurora borealis*, and the electric fluid, to be matter of the same kind. He supports this opinion by many strong arguments, which are found in his ingenious essay on the subject. According to his hypothesis, it would follow, that the tails are hollow, and there is every reason to suppose this, from the scarcely perceptible diminution of the lustre of the stars seen through them. He supposes that the electric matter, which continually escapes from the planets, is brought back by the assistance of the comets.

But much is yet to be known on this subject. Objections may be made to his hypothesis, although so ingeniously supported. According to the opinion of Kepler, the rays of the sun carry away some gross parts of the comet, which reflect other rays of the sun, and give the appearance of a tail.

CHAPTER IX.

CONSIDERATIONS ON THE SOLAR SYSTEM AND FIXED STARS.

159. MANY of the principal phænomena have now been examined, and the chief steps gone over, by which we arrive at the true arrangement, and motions of the bodies, that, on first viewing the heavens, are considered as all placed in the imaginary concave surface. The true motions have been distinguished from the apparent, and the magnitudes of the sun, moon, and planets have been ascertained, as also their situation with respect to the planet on which we live. This arrangement, that, with reference to the Sun, ought strictly to be called the Solar, is usually called the Copernican system. To give due honour to the memory of the discoverer, this name ought to be preserved; but, in retaining it, especial care should be taken, that the name attached may not occasion it to be ranked as a system of conjecture. It is not a system of hypothesis, but the system of nature.

160. The next steps in the science are the considerations of those observations, by which the motions of the celestial bodies may become more accurately known. An accurate knowledge of the laws of their motions is necessary to point out their places at any future period, and predict those phænomena which are a source of delight to the learned, and of fear to the ignorant. Long since mankind applied the motions of the celestial bodies to assist the sciences of Geography and Navigation. In more modern times it has been found, that the improved state of these

sciences requires a most accurate knowledge of the places and motions of those bodies. This has called the attention of astronomers and mathematicians to more particular exertions, and we, probably, owe thereto many most valuable discoveries, which, although not magnificently striking, are such as the mind must dwell upon with much pleasure; and which, perhaps, without the motive of utility, the love of science might not have investigated. Before we proceed to these parts, let us take a short review of the Solar or Copernican system, and of some circumstances connected therewith, which, if not equally certain, are many of them highly probable.

161. The earth, a spherical body of vast magnitude, when measured by our ideas, revolves on an axis in $23^h\ 56^m$, successively exposing its different parts to the light and influence of the sun, about which it moves annually in an orbit nearly circular, to which its axis is inclined at an angle of $66^\circ\ 32'$, and by its inclination causing the changes of seasons. It is attended by the moon, a spherical body, the magnitude of which is $\frac{1}{64}$ that of the earth, and which, moving round the earth in a month, is carried together with it annually about the sun The moon, by affording light during the absence of the sun, and by moving the waters of the ocean, is of great utility to the inhabitants of the earth. Yet we must not judge that for these causes solely the moon was formed: doubtless much weightier causes lie hid in the counsels of the Almighty, some of which at a future day it may be permitted to man to know. We perceive that in many respects it differs from the earth: it revolves about its axis in a much longer period: it is nearly destitute of an atmosphere: the irregularities of its surface are much greater than those of the earth. and probably it has no fluids on its surface similar to our own.

162. But the planets which revolve about the sun, we may consider as serving the same ends in the creation, as our earth.

In them we contemplate the noble spectacle of ten great bodies, revolving together with the earth round the sun, at different distances, and in different periods, but preserving a certain relation between their periods and distances. Of these Ceres, Pallas, Juno, and Vesta, are far less than the earth, but perform their revolutions round the sun, by precisely the same laws as the other planets.[a] Mercury and Mars are also less than the earth, and Venus nearly equal. Jupiter, Saturn, and the Georgium Sidus are considerably larger. Jupiter has four bodies carried with him round the Sun, and, as far as we can judge, subservient to the same ends as our moon. Saturn has seven, besides a double ring of stupendous dimensions; of the use of this, our limited knowledge will not permit us to judge; we can only perceive that it must, by its light, be most grateful to the inhabitants of the planet. The Georgium Sidus, so lately pointed out to our view, although in surface sixteen times, and in magnitude sixty-four times, larger than our earth, has six satellites visible to us, and their number will probably be increased with the goodness of our telescopes.

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[a] The very small magnitudes of the new planets Ceres and Pallas, and their nearly equal distances from the sun, induced Dr. Olbers, who discovered Pallas in 1802 nearly in the same place where he had observed Ceres a few months before, to conjecture that they were the fragments of a larger planet which had, by some unknown cause, been broken in pieces. It follows from the law of Gravity, by which the planets are retained in their orbits, that each fragment would again, after every revolution about the sun, pass nearly through the place in which the planet was when the catastrophe happened, and besides the orbit of each fragment would intersect the continuation of the line joining this place and the sun. Thence it was easy to ascertain the two particular regions of the heavens through which all these fragments would pass. Also by carefully noting the small stars thereabout, and examining them from time to time, it might be expected that more of the fragments would be discovered.—Mr. Harding discovered the planet Juno in one of these regions, and Dr. Olbers himself also, by carefully examining them from time to time, discovered Vesta.

163 In all these bodies, judging from analogy, probably the same admirable varieties of animate and inanimate beings are exhibited which we behold on this earth We know that several of them revolve on their axes, that their times of revolution are not very different from that of the earth, and that they are surrounded by atmospheres. Mars and Saturn have nearly the same variety of seasons as the earth. Jupiter considerably less. Venus has an atmosphere and mountains, and revolves on an axis.

We have no argument against the planets being inhabited by rational beings, and consequently by witnesses of the Creator's power, magnificence, and benevolence, unless it be said that some are much nearer the sun than the earth is, and therefore must be uninhabitable from heat, and those more distant from cold. Whatever objection this may be against their being inhabited by rational beings, of an organization similar to those on the earth, it can have little force, when urged with respect to rational beings in general. But we may examine, without indulging too much in conjecture, whether it be not possible that the planets may be possessed by rational beings, and contain animals and vegetables, even little different from those with which we are familiar.

164. On our earth the influence of the sun causes the heat of summer, and, from its absence, the cold of winter takes place; but is the sun the principal cause of the temperature of the earth? We have reason to suppose that it is not. The mean temperature of the earth, at a small depth from the surface, seems constant in summer and in winter, and is probably coeval with its first formation. The sun, by its influence, appears only to change the temperature at its surface, where heat is accumulated on account of the matter of the earth not suffering a farther transmission: this heat disappears in a variety of ways, by forming vapour, and so becoming latent, by being conducted to

the adjacent bodies, by coming into contact with cold air, &c., so that when the sun in winter remains only for a short time above the horizon, shines through a denser medium, and more obliquely, the consumption of heat is greater than the supply, and the cold of winter comes on We may also suppose that the matter of heat does not actually pass to us from the sun, but is only extricated, as it were, by his influence from substances in which it is compounded; for otherwise the temperature of the earth, either at the surface, or at a small depth from it, must be continually increasing, and that increase in a few years become sensible; since we know of no way for the heat which assists vegetation, which unites with fluids, &c. to pass off from the earth again Besides heat seems to exist in a state of combination in such profusion, that it requires only to be decomposed to answer every purpose. Is it not then unnecessary to have recourse to a continual supply from the sun, and may we not conceive, with some degree of probability, that in all the planets of our system, the temperature may be such as not to be inconsistent with a creation of animals and vegetables not very dissimilar to our own? And this, without appearing to limit the diversity of works in the universe, which we certainly are not authorized to do, for, wherever our senses or the deductions of reason can reach, we are sure of finding endless variety

165 At the planet Mercury, the direct heat of the sun, or power of causing heat, is six times greater than with us If we suppose the mean temperature of Mercury to be the same as of the earth, and the planet to be surrounded with an atmosphere, denser than that of the earth, less capable of transmitting heat, or rather the influence of the sun to extricate heat, and at the same time more readily conducting it to keep up an evenness of temperature, may we not suppose the planet Mercury fit for the habitation of men, and the production of vegetables similar to our own?

At the Georgium Sidus, the direct influence of the sun is 360 times less than at the earth, and the sun is there seen under an angle not much greater than that under which we behold Venus, when nearest Yet may not the mean temperature of the Georgium Sidus be nearly the same as that of the earth? May not its atmosphere more easily transmit the influence of the sun, and may not the matter of heat be more copiously combined, and more readily extricated, than with us? Whence changes of seasons similar to our own may take place. Even in the comets we may suppose no great change of temperature takes place, as we know of no cause which will deprive them of their mean temperature, and particularly if we suppose, that on their approach towards the sun, there is a provision for their atmospheres becoming denser. The tails they exhibit, when in the neighbourhood of the sun, seem in some measure to countenance this idea.

We can hardly suppose that the sun, a body three hundred times larger than all the planets together, was created only to preserve the periodic motions, and give light and heat to the planets. Many astronomers have thought that its atmosphere is only luminous, and its body opaque, and probably of the same constitution as the planets. Allowing therefore that its luminous atmosphere only extricates heat, we see no reason why the sun itself should not be inhabited.

166. Our knowledge of the fixed stars must be much more circumscribed than of the planets; we can, however, ascertain enough to be assured that our system is a portion of the universe most minute indeed The fixed stars, we have seen, are at immeasurable distances from us, at distances compared with which the whole solar system is but a point. Their diameters are less than we can measure, yet their light is more intense than that of the planets. We conclude, therefore, that they are self-shining bodies, and, according to a high degree of probability, like our

sun, the centres of planetary systems Admitting this, the multitudes of fixed stars, that may be discovered with the most inferior telescopes, shew us an extent of the universe, that our imagination can scarcely comprehend, but what is even this, compared to the extent that the discoveries and conjectures of Herschel point out?

167 We cease to have distinct ideas, when we enumerate by ordinary measures the distances of the fixed stars, and we require the aid of other circumstances to enable us to comprehend them. Thus, we compute that the nearest of the fixed stars is so far distant, that light will take above a year in coming from the star to the earth; that the light of many telescopic stars may have been many hundred years in reaching us, and still farther, that, according to Dr. Herschel, the light of some of the nebulæ, just perceptible in his forty-feet telescope, has been above a million of years on its passage.

168 We know, from the eclipses of Jupiter's satellites, that the velocity of light is so great, that it takes only about eight minutes in travelling from the sun to the earth, while the earth itself, moving with its velocity of nineteen miles in a second, would take nearly two months to pass over the same space We also know, as will be explained farther on, that the light of the fixed stars moves with the same velocity as the reflected light of the sun Hence, as we are certain that the distance of the nearest of the fixed stars exceeds 80,000 times the distance of the sun from the earth, the distance of the nearest star is such, that light must be above 400 days in passing from it to the earth.

169. The limit of the distance of the nearest fixed star may be considered as well ascertained, but any thing advanced with respect to the distances of the others, must be in a manner conjectural.

The brighter fixed stars have been supposed to be nearer to

us than the rest Besides their superior lustre leading to this conclusion, many of them were discovered to have small motions called *proper motions*, that only could be explained by supposing them to arise from a real motion in the stars themselves, or in the sun and solar system, or from a motion compounded of both these circumstances.

Now whichsoever of these suppositions was adopted, it was reasonable to suppose, that the cause of the smaller stars not appearing to be affected, could only arise from the greater distance of those stars. However it is now ascertained that some of the smaller stars appear to have proper motions, much greater than those of the brightest stars.

Hence conclusions deduced from the proper motions of the bright stars, respecting the relative distances of those stars must tend to weaken conclusions that might be deduced from their brightness and apparent magnitudes.

There is a double star of the sixth magnitude, the 61st star of the constellation of the Swan, which consists of two stars, within a few seconds of each other. Both of these stars are moving nearly at the same rate, at the rate of about 6″ in a year It is likely they are also moving about their common centre of gravity At present they preserve nearly the same distance from each other This proper motion is far greater than has been observed in any of the brighter stars, or indeed in any star. It might be supposed, on this account, that these stars (61 Cygni) are nearer to us than the brighter stars. To ascertain this point, I have made observations of the zenith distances, at the opposite seasons, to endeavour to discover any sensible parallax in these stars But there appears to be no sensible parallax. Mr. Bessel has compared these and some of the neighbouring stars by observations on the right ascensions, and found no sensible parallax. Still the arguments formerly adduced, for the brighter fixed stars being nearer to us, are considerably weakened by the great proper motions observed in some of the smaller stars.

The star 40 Eridani has a proper motion of about four seconds in a year. The annual proper motion of Arcturus is about two seconds.

In many of the stars there is no proper motion perceptible.

Besides the proper motions, it has been remarked by Dr Herschel, that in several instances, the line joining two stars very near together, changes its position.

This is in some cases explained by a proper motion in the brighter star, in other cases it seems to indicate the revolution of one star round another The double star Castor is a striking instance. during the last fifty years, the line joining the two stars, which are about five seconds asunder, has had a motion of rotation at the rate of about a degree in a year, while the interval between the stars has remained nearly the same. Of the three circumstances which explain the apparent motion of a star, that which supposes it to arise from a combination of the motion of the solar system and of the star is most probable. The sun and nearest fixed stars are probably all in motion round a centre, the centre of gravity, perhaps of a nebula, or cluster of stars, of which the sun is one, and the milky way a part, as Dr. Herschel supposes, while this nebula revolves with other nebulæ about a common centre.

170 The direction of the motion of our system cannot with certainty be ascertained, because, from the whole motion we observe in a fixed star, we have nothing to help us in assigning that which belongs to the sun. Dr Herschel has particularly considered this subject (Phil. Trans. 1805 and 1806), and has concluded that our sun is moving towards a point in the constellation Hercules, the declination of which is 40°, and right ascension 246°. His arguments are very ingenious, but there is necessarily so much hypothetical in them, that the mind cannot feel much confidence in his conclusion. That our system is in motion, there can be no doubt; the difficulty is to ascertain the

precise direction and velocity: and from the circumstances of the case, there seems to be little probability that the knowledge will ever be here attained to by man.

Dr. Herschel conjectures that the distances of the fixed stars are nearly inversely as their apparent magnitudes. From thence, and a train of ingenious reasoning, relative to the faintest *nebulæ* discoverable by his 40 feet telescope, he has concluded that the distances of these nebulæ are so great, that light issuing from them must have been two millions of years in reaching the earth. But the recent discoveries relative to the proper motions of the smaller fixed stars must, as has been said, in some measure weaken the conclusions formerly adopted respecting the relative distances of the fixed stars. The discoveries of Dr. Herschel have also made us acquainted with many nebulæ, which are not resolvable into stars, but apparently formed of luminous matter, gradually condensing, by the principle of universal attraction, into masses, as if about to form the suns of future systems. Distant ages only can appreciate these conjectures.

CHAPTER X.

OBSERVATIONS FOR ASCERTAINING THE DECLINATION—DISTANCE OF THE POLE FROM ZENITH—OBLIQUITY OF ECLIPTIC—RIGHT ASCENSION.

171 PREVIOUSLY to a more minute statement of the motions of the celestial bodies, it will be necessary to give some account of the nature of the principal observations, by which these motions are ascertained, and of the instruments by which the observations are made

The most important observations, and which admit of the greatest accuracy, are those for the declination and right ascension Having obtained the declination and right ascension, or the position with respect to the celestial equator, we can by spherical trigonometry obtain the longitude and latitude, or the position with respect to the ecliptic The latitude and longitude of any of the bodies of the solar system, as they would be observed from the centre of the earth, are called their *geocentric* latitude and longitude

The tables give the distance of the body from the sun, and its place, as seen from the sun, or its *heliocentric* longitude and latitude, from whence we can compute its geocentric latitude and longitude, and compare them with those observed.

172 The *declination* of an object is best found by observing its distance, when on the meridian, from the zenith or from the horizon Either of these distances being found, if we previously

know the distance of the pole from the zenith of the place, we find by addition or subtraction the distance of the object from the elevated pole, and consequently its declination Thus let HESZPO (Fig 24) represent the meridian, HO the horizon, E, S, Z, P the places of the equator, object, zenith, and pole; we observe HS or SZ in the way that will be presently shewn; and if we have ZP, we easily obtain SP, the polar distance, or ES the declination There is an advantage in using the polar distance instead of the declination, because in the former there is no ambiguity, but when the declination is used, it is necessary to note whether it be north or south. Accordingly many astronomers use in their catalogues of stars north polar distance instead of declination: thus, if the declination be 20° S. its north polar distance is 110°

It must be understood, that the zenith distance, or altitude observed, is to be corrected by refraction, and by some other small quantities, as sometimes by parallax, (to reduce it to what would be observed at the earth's centre), by aberration of light and nutation of the earth's axis, which corrections will be explained hereafter, and are usually obtained from tables.

173. The distance of the zenith from the pole is found by observing the zenith distance of a star that does not set, when on the meridian above and below the pole: thus let ZR (Fig 24) be the zenith distance, corrected for refraction, of a star, when above the pole, and Zr the zenith distance, corrected for refraction, when below the pole, then $ZP = ZR + RP$ and $ZP = Zr - rP = Zr - RP$. Hence $2ZP = ZR + Zr$. PO, as we have seen, (Art 39), is equal to the latitude of the place, and therefore ZP is the complement of latitude. Hence the observations for ascertaining the distance of the pole from the zenith, give us the latitude of the place of observation

By repeating this observation for the same star, and for different stars, a great many times, the distance of the pole from

the zenith may be had with great exactness, that is, with good instruments, to less than a second. This element being once established, we are enabled, as stated above, to obtain by observation the declination or polar distance of any celestial phænomenon. It is necessary in this mode of observation to know with some degree of exactness when the object is at the meridian; this will be explained hereafter.

174. The declination of an object may also be obtained by observations made out of the meridian: Thus, if we observe the distance from the zenith ZF, and azimuth FZP, and know ZP, we obtain, by the solution of the spherical triangle ZPF, PF the polar distance. or if ZF be observed, and we know ZP and the angle FPZ (known by the distance in time of the body from the meridian) we can compute FP but the instruments, required to make these double observations, are too complicated in their construction to be used with advantage in making very accurate observations; to which may be added the inconvenience in computing the effects of parallax and refraction. Refraction and parallax, when the body is in the meridian, only affect the declination, they affect both right ascension and declination, when the body is not in the meridian.

175. There is also a method of obtaining both the declination and right ascension at the same time, by an instrument called an equatoreal instrument.[a] Although this, when well executed, is a very valuable instrument, yet being complicated, and admitting of less precision for the declination than the method described, it will not be necessary to dwell upon it here It may, however, be proper to remark, that sometimes objects of faint light, such as comets, pass the meridian in day light, and cannot be then observed, for these an equatoreal instrument is

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[a] Professor Vince's Practical Astronomy See also a description of one made for Sir George Shuckburgh, Phil Trans 1793.

very convenient A very advantageous method of ascertaining the right ascension and declination of such objects, is by observing, with a micrometer, the differences of right ascension and declination between the object and a neighbouring fixed star. the position of the latter is previously known, or may be observed at leisure.

176. In computing the longitude and latitude of an object, from knowing its right ascension and declination, we use the obliquity of the ecliptic The obliquity of the ecliptic is found by observing the greatest declination of the sun If many declinations be observed when the sun is near the solstice, each of these may by a small correction be reduced to the declination at the solstice, and the mean of all taken. The advantage of this is, that the declination observed within a few days of the solstice may easily be reduced to the greatest declination, without knowing with great accuracy the right ascension of the sun The summer solstice is to be preferred to the winter one, on account of refraction being more uncertain at lower altitudes.

177 To ascertain the right ascension of an object, it is necessary to find the arch of the equator intercepted between the first point of Aries and a secondary passing through the object: for this purpose we make use of a portion of duration, called sidereal time. The whole concave surface revolves uniformly in twenty-four hours of sidereal time (Art. 14), and any portion of the equator is measured by the sidereal time elapsed between the passages of its extremities over the meridian: thus the extremities of an arch of 15° pass the meridian at an interval of one hour. Hence we conclude, that the difference of right ascension of these extremities is 15° or one hour so that the right ascension of any object is measured by the portion of sidereal time elapsed between the passages or transits of the first point of Aries (the intersection of the ecliptic and equator), and of the object over the meridian. Hence if a clock be adjusted

to shew twenty-four hours during the rotation of the concave surface, and commence its reckoning when the first point of Aries is on the meridian, it will shew the right ascension of all the points of the concave surface on the meridian at any time; and all that is necessary to ascertain the right ascension of any object, is to observe the time shewn by the clock when that object passes the meridian. This time is the right ascension, and being multiplied by 15, gives the right ascension in degrees, &c.

The instrument by which the time of the transit over the meridian is accurately observed, and the manner of observing it, will be presently explained.

178. The intersection of the ecliptic and equator not being marked on the concave surface, we must, for regulating the clock, make use of some fixed star, the right ascension of which is known: the clock may be put nearly to sidereal time, and the exact time being noted when a star, the right ascension of which is known, passes the meridian, the error of the clock will be known. Thus if the clock shew $1^h\ 15^m\ 14^s$, when a star, the right ascension of which, is $1^h\ 15^m\ 10^s$, passes, the error of the clock will be 4^s, and every right ascension observed must be corrected by this quantity.

179. It is evident then, that the right ascension of some one star being known, the right ascensions of the rest may be obtained with much facility. The method which follows, has been used by Mr. Flamstead, and by astronomers in general, to obtain the right ascension of α Aquilæ.

When the sun between the vernal and autumnal equinoxes has equal declinations, its distances in each case, from the respective equinoxes, are equal. We can ascertain when the sun has equal declinations, by observing the zenith distances for two or three days, soon after the vernal equinox, and for two or three days about the same distance of time before the autumnal,

and then, by proportion, ascertain the precise time when the declinations are equal. at these times also we can ascertain, by proportion, the differences of the right ascension of the sun and some star, by observing the differences at noon for two or three days Let

E = the right ascension of the sun, soon after the vernal equinox, then $180° - E$ = the right ascension before the autumnal, when it has equal declination.

A = the right ascension of the star in the former instance

$A + p$ = the right ascension in the latter

We obtain by help of observations $A - E$ and $(180 - E) - (A + p)$. Let these differences of right ascension be D and D', that is,

$$A - E = D$$

and $(180 - E) - (A + p) = D'$. From which we can determine E and A. For, adding these equations $180° - 2E - p = D + D'$ or $E = \frac{180° - (D + D') - p}{2}$ and thence $A = D + E$ is known The value of p arises from the change of right ascension of the star in the interval between the times of equal declinations, and is therefore known from the tables of precession and aberration, &c

This kind of observation may be repeated many times for the same star between two successive equinoxes, and likewise in different years; and, by taking a mean of many results, great precision will be obtained

The advantage of this method is, that the sun's zenith distance being the same at the two times of observation, probably, any error in the instrument will equally affect each zenith distance, and therefore we can exactly find when the declinations are the same, although we were not able to observe the declination itself with the greatest accuracy.

180. The construction of clocks for astronomical purposes

has arrived at such a degree of perfection, that, for many months together, their rate of going can be depended on, to less than a second in twenty-four hours. This accuracy has been obtained by the nice execution of the parts, in consequence of which the errors from friction are almost entirely avoided, and, by using rubies for the sockets, and pallets, where the action is most incessant, the effect of wear is almost entirely obviated. But the principal source of accuracy is the construction of the pendulums, which are so contrived, that even in the extremes of heat and cold they remain of the same length. This is generally effected by a combination of rods of two different metals, differing considerably in their expansive powers. They are so placed as to counteract each other's effects on the length of the pendulum. Formerly brass and steel were used, the former expanding much more by heat than the latter. In this construction nine rods or bars were placed by the side of each other, and the pendulum, from its appearance, was called a gridiron pendulum. A composition of zinc and silver is now frequently applied instead of brass, on account of its greater expansion, by which five bars are made to serve. Other constructions are also used, for preserving the same length in the pendulum, but not so commonly.

181. A clock of this description is absolutely necessary for an observatory. It is regulated to sidereal time, and the hours are continued to twenty-four, beginning when the vernal intersection of the ecliptic and equator is on the meridian; and not like common clocks, at noon. But however well executed the clock may be, it is depended on only for short intervals; the time it shews being examined by the transit of fixed stars, the right ascensions of which have been accurately settled. For this purpose the right ascensions of thirty-six principal stars were determined with great exactness by Dr. Maskelyne. Several of these may be observed every day, each observation

pointing out the error of the clock; and the mean of the errors will give the error more exactly. Nothing more then is necessary for determining the right ascension of a celestial object, than to observe the sidereal time of its transit by the clock: that time, being corrected, if necessary, by observations of the standard stars, is the right ascension.

CHAPTER XI.

METHODS OF ASCERTAINING MINUTE PORTIONS OF CIRCULAR ARCHES —ASTRONOMICAL QUADRANT—ZENITH SECTOR—CIRCLE—AND TRANSIT INSTRUMENT—METHODS OF FINDING THE MERIDIAN.

182. As the arches or limbs, as they are called, of astronomical instruments, are seldom divided nearer than to every five minutes, it is necessary briefly to explain the methods by which smaller portions may be ascertained: there are three methods now principally used, 1. by a *vernier*; 2. by a *micrometer screw*; 3. by a *microscope*.

183. The first method is of more general use than the other two, and is applied to a great variety of philosophical instruments. It is named after its inventor. It will be easily understood by an instance. Let the arch *lt* (Fig. 25) be divided into equal parts, *lh*, *hm*, *mn*, *np*, &c. each 20′, and let it be required to ascertain smaller portions, for instance, the distance of P from *p*A. Let another circular-arch, called the vernier, 7° long, slide upon the arch *lt*, and let it be divided into twenty equal parts, that is, each part $= \frac{7 \times 60}{20} = 21'$. If these parts be *bc*, *cd*, *de*, &c. then the division *d* coinciding with the division *m*, the division *c* will be $(21' - 20')$ or 1′ beyond the division *n*; the division *b* 2′ beyond the division *p*, &c. So that in this way we can ascertain portions of 1′, 2′, &c., although the arches themselves are divided only into portions of 20′. To apply this, suppose it were required to ascertain the distance of P from *p*A:

let the vernier be slided till its commencement b coincides with P, then it will be seen what division of the vernier coincides with a division of the limb the divisions of the vernier are numbered from its beginning 0, 1, 2, 3, &c The number of the coinciding division of the vernier will, it is manifest, shew the distance of the commencement of the vernier from the division on the limb or PA In the application of this instrument to astronomical purposes, the vernier is so attached, that its commencement or point of Zero, as it is called, is always brought by the process of making the observation to the point from which the reading is to be made. In other applications, in the barometer, for instance, the commencement of the vernier is to be moved to that point

This method of ascertaining the extent of small arches is more frequently used, where the measurement is only to be made to the nearest minute, but it may be readily applied to ascertain much smaller portions Thus, if the limb be divided into portions of 20′, and a vernier = 19° 40′ be divided into sixty parts, each of these parts will be 19′ 40″, and therefore an interval on the limb exceeds an interval on the vernier by 20″, and so a space of 20″ is ascertained.

Again, if the limb be divided into parts of five minutes each, and a vernier = 4° *55′* be divided into sixty parts, each of these parts will be 4′ *55″*, and therefore an interval on the limb exceeds an interval on the vernier by *5″*.

184. To ascertain still smaller portions, a micrometer screw answers better; which is a very fine screw, requiring to be well executed, so fine that the interval between two threads is sometimes only the $\frac{1}{10}$ of an inch A circular head is fixed to this screw. This head is divided into equal parts, the whole being the number of seconds answering to the interval between the threads. This screw is attached to the part of the instrument carrying the point P, (Fig 25) moves at right angles to the ra-

dius, and is to be turned till the point P coincides with A. The number of complete revolutions, and the part of a revolution will give the seconds in PA.

185 The above is very convenient and exact, but it yields to Mr. Ramsden's method, by microscopes, in which the image of the arch and of the point P is formed in the focus of a compound microscope, the axis of which is perpendicular to the plane of the limb, a wire in the focus of this microscope is moved by a micrometer screw, and made to pass successively over the images of the points P and A, and the motion of the screw shows the interval. The advantage of this method arises from the distinctness and magnitude of the image in the microscope. The exact coincidence also of it with the wire in the focus assists much the accuracy of observation. This application of microscopes is justly considered as a very valuable improvement in astronomical instruments

THE ASTRONOMICAL QUADRANT AND CIRCLE

186 The quadrant for measuring zenith distances, is moveable on a vertical axis, or fixed to a solid wall in the plane of the meridian. In the latter case it is called a mural quadrant. The telescope, which is moveable about the centre of the quadrant, has an index, usually a vernier, fixed to it, and moving on the divided arch of the quadrant. The plane of the quadrant being perpendicular to the horizon, and in the same vertical circle as the object, the telescope is moved till the object appears near the centre of the field of view, touching or bisected by a wire, placed in the principal focus of the telescope, parallel to the horizon, or at right angles to the plane of the quadrant. The arch then between (*o*) on the vernier, and the lowest point of the quadrant from which the divisions commence ((*o*) of the arch), shows the zenith distance, provided the radius passing through (*o*) of the

arch be vertical, and provided also that the line of collimation of the telescope be parallel to the radius passing through (*o*) of the vernier. The methods of ascertaining the exact place of the arch, pointed out by (*o*) on the vernier, have been shewn in Art 183, &c The radius passing through (*o*) of the arch is generally made vertical, by help of a plumb line The plumb line bisecting a point near the centre of the quadrant, is made to bisect another point on the arch, by moving the quadrant in its own plane These two points are placed by the maker, parallel to the radius, passing through (*o*) of the arch

187 *The line of collimation* of a telescope, is the line joining the centre of the object glass, and the place of the image in the principal focus this is the true direction of the object, in which it would be viewed by the naked eye. Hence it is evident that this line ought to be parallel to the radius passing through (*o*) on the vernier, that the angle measured by the distance of (*o*) on the vernier from (*o*) on the quadrant, may shew the angle contained by a vertical line, and the line of direction passing through the object, which angle is equal to the zenith distance of the object.

Thus OP (Fig 26) represents the plumb line passing over two points. The line which joins these points is parallel to the radius CL, passing through (*o*) of the arch. The dotted line DI is the line of collimation, parallel to the radius CV passing through (*o*) of the vernier. LV measures the zenith distance of the object, the image of which is at I. The vernier being fixed to the telescope, the radius CV, while the telescope moves, always preserves the same relative position to the line of collimation. The position of the line of collimation must always be scrupulously attended to, and, if erroneous, must either be adjusted by moving the wire in the focus of the telescope, or the error allowed for, the latter is generally better, when the error amounts only to a small quantity

188 To enable the observer to ascertain the error of the

line of collimation in those quadrants that move on a vertical axis, the arch is continued several degrees beyond (*o*) (Fig. 26) as PS, and the zenith distance of the same object is to be observed with the arch of the instrument facing different ways Thus, when a star near the zenith is observed, let CT (Fig 27 1) be the radius, parallel to the line of collimation of the telescope, CV the radius passing through (*o*) on the vernier Then LV is the arch read off or observed; which is too little by TV Let the quadrant be moved on its vertical axis half round the position of the above lines will be as in Fig 27. 2. Then that the telescope may be directed to the same star, it must be moved over the arch TT′, till it is parallel to its former position CT (Fig. 27. 1) so that L′T′ = LT. The point V is transferred by the motion of the telescope to V′, &c. The arch now measured is V′L′ too great by V′T′ = VT Hence 2VT (double the error of the line of collimation) = difference of the zenith distances of the same star, observed in the two positions of the quadrant

189. This method cannot be adopted for mural quadrants; for these another instrument is necessary A zenith sector is used, at the Royal Observatory at Greenwich, to ascertain the errors of the lines of collimation of the mural quadrants. This instrument is principally composed of an arch of a circle of a long radius, fixed to a telescope of the same length, passing through the centre and middle of the arch The instrument is suspended vertically: the telescope can be moved (the arch being fixed to it) a few degrees on each side of the vertical line, so as to observe stars within a few degrees of the zenith. A plumb line suspended from the centre of the instrument, and passing over the arch, shews the angle between the point of (*o*) at the telescope and the vertical line This instrument is capable of having its face turned both east and west; therefore, if observations of the same stars be compared in both circumstances,

the error of the line of collimation will be had as in a quadrant moving on a vertical axis · consequently, if observations are made at the same time with a mural quadrant, and compared with the observations made with the zenith sector, the error of collimation of the mural quadrant will be ascertained. The error of the line of collimation of a quadrant is not much liable to vary.

190. The brief account of astronomical instruments, here given, is only intended to contain enough to afford such a general conception of their uses, that the observations mentioned may be understood. We may refer to Professor Vince's Practical Astronomy for a more particular account of astronomical instruments · where the astronomical quadrant, its uses and adjustments, are minutely described; as also the zenith sector, used at the Greenwich Observatory A zenith sector, constructed for the purpose of the great trigonometrical survey, now carrying on in England, is particularly described in Phil. Trans for 1803.

191. The radius of each of the Greenwich mural quadrants is eight feet, and their arches are divided into parts of 5′ each, and, by means of the micrometer screw, angles are easily ascertained to seconds of a degree, a second of a degree in these instruments is $\frac{1}{2000}$ of an inch nearly. They were divided by Mr. Bird, the first artist who attained a great degree of perfection in the divisions of large instruments, and who, perhaps, has not since been excelled. The quadrant facing the east, with which all observations to the south of the zenith are made, is superior to the other. This instrument will always be celebrated in the history of astronomy; with it and the transit instrument were made the numerous and valuable observations of Dr Bradley, and of the late Dr Maskelyne, that were continued through a space of near sixty years, and which, by the concurrent approbation of astronomers and mathematicians of all countries, have been adopted as the basis of those improvements, that have advanced astronomy to its present degree of perfection.

192 The present state of Astronomy requires the utmost perfection in the instruments. those for zenith distances ought to admit of a degree of precision, enabling us to observe the zenith distance with certainty to two or three seconds; so that, by taking a mean of a number of observations, we may have the zenith distance to 1″ or less. This accuracy is necessary to ascertain the motions of the moon to the requisite exactness, and to determine the effects of the mutual disturbances of the system, many of which, at their maxima, amount only to a few seconds

193 A quadrantal arc appears, at first view, to be all that is necessary for observing the zenith distance of a celestial object; a larger portion of a circle seems useless, and, by confining the instrument to a quadrantal arc, we may have the advantage of a much longer radius, but experience has shewn that quadrants yield in accuracy to complete circles.

The astronomical quadrants will probably soon be entirely superseded by circles.[a] There is reason to suppose that much greater accuracy will be obtained by the latter than by the former, even when the diameter of the circle is far less than the radius of the quadrant. Nearly a century ago, an entire meridian circle had been executed · yet there was nothing in this that can lessen the credit due to the late Mr Ramsden, who above twenty years ago, proposed to substitute the circle for the quadrant. The astronomical circle serves exactly the same purposes as the quadrant Mr. Ramsden made his first circle, five feet in diameter, for the Observatory at Palermo Since that time several circles have been constructed by other artists, particularly by Mr. Troughton. The Observatory of Trinity College, Dublin, possesses a most superb circle eight feet in diameter.

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[a] For an account of astronomical circles, see Mr. Vince's Practical Astronomy and Phil Trans 1806, part 2

This instrument was begun by Mr Ramsden, and finished by his successor, Mr. Berge,[a] and has been in use since the latter end of 1808

194 The advantages, proposed to be obtained by the astronomical circles, are

1. The telescope is fixed to the circle, and the whole moves together on an axis, which is a great advantage, as all imperfections of centre work, danger from the telescope bending, or the centre work wearing, are avoided.

2 All parts of the instrument can be readily exposed to the same temperature

3 The instrument balances itself.

4 All imperfections of the divisions are readily discovered, as the same angles can be observed on different parts. also the instrument can be much more accurately divided in consequence of the person, who divides, being enabled to examine opposite points [b]

5. One of the greatest advantages of our instrument is the

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[a] This instrument was originally begun by Mr Ramsden, directed by my predecessor, Dr Usher, at the desire of the College, about the time when it was first in contemplation to substitute circles instead of quadrants After furnishing one of five feet in diameter, for the Observatory at Palermo, Mr Ramsden engaged to finish one of ten feet for our Observatory, but most tedious delays, much indeed to be regretted, intervened. After having partly executed one of ten feet, he rejected it for one of nine feet, and this, after the circle itself was actually divided, was laid aside for the present one of eight feet, which he left to his successor, Mr. Berge, to divide and finish, and share the credit due for the execution of such an instrument In speaking of this instrument, the liberality of the Provost and Senior Fellows of the University of Dublin ought not to be passed over. With their usual zeal for the promotion of knowledge, they spared no expense to obtain, for their Observatory, the first instrument of its kind, shewing on this, and on all occasions, where the interest of science is concerned, an example well worthy of imitation

[b] Mr. Troughton has described his very ingenious method of dividing Astronomical Instruments, in the Phil. Tran 1809, Part 1.

facility with which the error of the line of collimation ma
found, by observing a star at any distance from the zenit
the face of the instrument turned different ways Thus
nith sector, which would be absolutely necessary for a
quadrant, can be dispensed with.

6. The method of reading off by compound micros
which Mr Ramsden has adapted to this instrument, is gr
preferable to the methods by the vernier or micrometer
Art. 183 and 184.

In our circle[a] three microscopes are used; one is pl
opposite to the lowest part, and two opposite to the horiz
diameter; by which the same angle may be read off on
different parts of the circle, and thus the errors from diff
of temperature may be obviated, and the effect of any inac
in the divisions considerably lessened This circle was ori
tended for meridional observations, but, on account of the
lity of the vertical axis, arising from the firmness of its su
it may with much advantage be used for a few minutes b
and after the passage of the object over the meridian, th
time of making the observation can be readily noted by
transit clock, and thence the correction computed for red
the observation to what it would have been, had it been
on the meridian

In the year 1812 a mural circle of exquisite workman
six feet in diameter, made by Mr Troughton, was placed i
Observatory at Greenwich. This instrument is now used
stead of the mural quadrants. It gives north polar dist
and not being capable of being reversed by a vertical axi
the observations are corrected by means of the north polar

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[a] Four microscopes placed at the extremities of the vertical and horizont
meters, would have afforded greater accuracy but a microscope opposite to
tom microscope would have been very inconveniently situated for use.

tances of certain standard stars, in a manner similar to that by which observations of the transit instrument are corrected for the error of the clock. (Vid. Art. 181.)

195. The observations made with the quadrant and circle being for the same purposes, an example from the latter will suffice.

Example. Aug. 23, 1808, at the Observatory of Trinity College, Dublin, observed, by the astronomical circle, the polar star above and below the pole,

ZD above	34° 53′ 10″,1	Barom. 29,99	Ther. 58°
below	38 18 59,1	- 29,97	- 67

Each of these zenith distances is from a mean of observations, made with the instrument facing east and west. To determine the zenith distance of the pole, and polar distance of the polar star.

	34 53 10″,1	38 18 59,1
Ref.	+ [a]39, 84	+ 44,47
	34 53 49, 94	38 19 43,57
	38 19 43, 57	
	2 | 73 13 33, 51	
	36 36 46, 75 zenith distance of pole or co-latitude	

This observation therefore gives the latitude of the observatory $= 53° \; 23' \; 13'', 3$.

Half the difference of the zenith distances $= 1° \; 42' \; 56''$,

[a] The above refractions are computed by a table, which is given in the Appendix. The results of my observations relative to refraction agree nearly with the French (M. Delambre's) Tables as far as about 80° Z. D. Nearer the horizon refractions become so uncertain, that observations cease to be of use.

81 = the observed polar distance of the polar star. This must be reduced to its mean quantity at some given time or epoch, as it is called, suppose, Jan. 1, 1813.

The change of N polar distance by the recession of the equator on the ecliptic (Art. 90,) from January 1, 1808, to January 1, 1813,) the change in a year being = — 19″, 50.)	— 1′ 37″, 50
By Precession from Jan 1, to Aug. 23	— 12, 50
Aberration of light - - -	+ 13, 38[a]
Nut. of the earth's axis from moon -	— 3, 32[b]
from sun -	— 0, 46[c]
Sum according to their signs -	— 2, 90

This is to be subtracted from the mean polar distance, Jan 1, 1808, to obtain the apparent polar distance, Aug. 23, 1808. Hence the mean polar distance, Jan 1, 1808, deduced from this observation is the sum of this quantity and the polar distance observed Therefore

1° 42 56, 81
+ 2, 90

1 42 59, 71
— 1 37, 50

1 41 22, 2 = mean polar distance of the pole star, Jan. 1, 1813

[a] General Tables of Aberration Professor Vince's Astron. vol i p 327.

[b] General Tables of Nutation, do vol ii. p 131

[c] Dr. Maskelyne's Tables Tab. 30 The aberration of light and its effects are explained hereafter Nutation has been mentioned, art 91. Solar nutation in NPD = — 0″, 48 sin. (2 lon. sun—R. A star)

By the mean of a number of observations it is 1° 41′ 21″,8
By the Greenwich circle it is 1° 41′ 21″, 7

TRANSIT INSTRUMENT.

196 The transit instrument, equally necessary in practical astronomy, as the quadrant or circle, is a telescope fixed at right angles to a cross axis. This axis is placed upon horizontal supports, upon which it turns. The instrument is to be so adjusted, that the line of collimation, when the telescope is turned with its axis, may move in the plane of the meridian. In the principal focus of the object glass are placed three or five wires, parallel to each other, and perpendicular to the horizontal axis. Another wire bisecting the field of view is also usually placed at right angles to these. The transit instrument in the Observatory of Trinity College, Dublin, is six feet long, the cross axis four feet, and there are five parallel and equidistant wires in the principal focus, and one at right angles to these.

In Fig. 28 the wires are represented.

To make the centre wire *Cd* move in the plane of the meridian, three adjustments are necessary.

1st. To make the axis level: this is done either by a spirit level or plumb line. 2dly. To make the line of collimation, that is, a line joining *Cd*, and the centre of the object glass, perpendicular to the axis: this is done as follows: let the image of a distant object be bisected by the middle wire, and then take the axis off its supports, and reverse it; if the image is then bisected, the line of collimation is exact, if not, half the error must be corrected by moving the system of wires, and half by moving one of the supports of the axis. There is a provision for both these motions. The axis being again reversed, will verify the adjustment. 3dly. The line of collimation is to be placed in the meridian: this is done by the help of a mark placed at a

considerable distance, for instance, at the distance of one or two miles, in the direction of the meridian of the centre of the instrument. The whole telescope is moved by moving one of the supports of the axis, till the middle wire bisects the image of the mark The meridian mark will serve to adjust the line of collimation, and indeed, in practice, the order of these adjustments should be reversed.

197. The use of the transit instrument is to determine the right ascensions of the celestial bodies, and also the mean and apparent time In observing the right ascension, the telescope is usually directed to the object, by help of a divided semicircle, placed at one end of the axis, on which an index attached to, and perpendicular to the axis, and also parallel to the line of collimation, moves, this index is to be set to the polar or zenith distance of the object, according as the semicircle shews polar or zenith distances

This being done, the time of passage of the object over each wire is noted by the clock, beating seconds and showing sidereal time, placed near the transit instrument. The mean of the observed times of passing each wire is to be taken to shew more accurately the time at the middle wire. The time of passing each wire may be observed with great accuracy, because the telescope magnifies the diurnal motion, so that at one beat of the clock a star may be observed on one side of the wire, as at *a*, and at the next beat, at *b* The eye is capable of pretty accurately proportioning the intervals *ac* and *bc*, so that the time may be noted to tenths of a second, and the mean from the five wires rarely deviates $\frac{2}{10}$ of a second of time from the truth, or $3''$ of a degree Thus right ascensions may be determined with nearly the same accuracy as zenith distances For, as has been already shewn, the time of the passage by the clock is the right ascension, provided the clock shews accurate sidereal time This is seldom the case, and ought always to be examined by

observing some of the thirty-six stars before mentioned, (Art 181) the right ascensions of which have been determined with great accuracy by Dr. Maskelyne

198. Example Observed at the Observatory of Trinity College, Dublin, Nov. 2, 1793, the transits over the meridian of

α Cygni at $20^h\ 34^m\ 15^s$, 72 sidereal time.

o Aquarii 21 52 29,89

Fomalhaut 22 46 4,65 the clock losing 1^s, 5 in 24 hours

To find the mean right ascension of *o* Aquarii

Mean A R Jan. 1, 1793,

	α Cygni	Fomalhaut
by Dr. Maskelyne's Catalogue	20 34 22 57	22 46 10,72
[a] Aber and precess. -	+ 1,53	+ 3,43
[a] Nutation -	0,00	— 0,85
App. A. R. -	20 34 24,10	22 46 13,30
Observed -	20 34 15,72	22 46 4,65
Clock slow -	8,38	8,65

Mean at $21^h\ 40^m$ slow 8^s,51

At 21 52 slow 8,52

Observed *o* Aquarii - -	21 52 29,89
Clock slow - - -	+ 8,52
Apparent A. R - -	21 52 38,41
Precess. from Jan. 1, 1790 -	— 11,87
[b] Aberration - - -	— 0,32
[b] Nutation - - -	+ 0,55
Mean A. R. Jan 1, 1790 -	21 52 26,77

[a] Professor Vince's Astronomy, vol. ii p 306, &c. Dr. Maskelyne's Tab. 17 and 18.

[b] By general tables referred to in Art. 195.

From Dr Bradley's observ - 21 52 26,8
—— Mr Pond's observ in 1816 21 52 26,6*

199. The transit instrument serves also for finding the mean, and thence the apparent time

If the sun, instead of appearing to move in the ecliptic with an unequable motion, appeared to move in the equator with an equable motion, in the period of its motion in the ecliptic, its return to the meridian would each day be later than the return of a fixed star, by $3^m\ 56^s$ nearly; and a clock put to twelve o'clock, when the sun was in the meridian, would, if rightly adjusted, always continue to shew twelve, when the sun, so moving, passed the meridian; and the time pointed out by the clock would be *mean time*

The distance of an imaginary sun, so moving in the equator, from the vernal equinox, is equal to the mean longitude of the sun, or its mean distance from the vernal equinox; and this distance, reduced into time, is the right ascension of the imaginary sun. The mean longitude of the sun is given in the Solar Tables for the beginning of each year, and the mean motion in longitude, between the beginning of the year and each day, is also given Whence the mean longitude is known, which reduced into sidereal time, at the rate of 15° for 1 hour, gives the right ascension of the imaginary sun, after being corrected, to reduce it to the true equinox. Hence, having the sidereal time, by a clock, or from the time shewn by a clock corrected by observing the transit of a star, the mean time is readily found. For, the difference between the imaginary sun's right ascension at noon (the mean longitude of the sun converted into time), and the given sidereal time, is the sidereal time from noon: this is to be reduced into mean time, by diminishing it in the proportion of $24^h : 23^h\ 56^m\ 4^s$, 1, or of 366 . 365 nearly. The mean

* This star, therefore, has no perceptible proper motion.

time being found, the apparent time will be had by applying the equation of time, which will be explained hereafter.

200 Example To find the mean time at Greenwich, Nov 8, 1808, at 20h 30m 7s, by the sidereal clock.

					N.
[a] Sun's mean long Jan 1, 1808.	9s	9°	58′	1″,1	337
Mean motion, Nov 8.	10	7	31	19	46
	7	17	29	20,1	383
Equat equinoxes in R A.				+ 11,1	
Mean long. at noon from true equinox.	[b] 7	17	29	31,2	
This reduced to time gives					
Sidereal time at noon	15h	9m	58s,1		
	20	30	7		
Interval from noon in sidereal time	5	20	8,0		
reduction to mean time		—	52,4		
Mean time	5	19	16,5		

For any other place, the change of the sun's mean longitude, according to the longitude of the place, must be allowed for. Thus, for the Observatory of Trinity College, Dublin, the sun's mean longitude is 1′ 1″, 6 greater than at Greenwich, or 4s, 16 in sidereal time.

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[a] Vince's Astronomy, vol 3, pp 1, 20, p 10 Table 8 Solar Tables.

[b] It is convenient to have Tables by which the mean longitude of the sun may be found at once in sidereal time. Such are Tables 19 and 20 in Dr. Maskelyne's Collection.

METHODS OF FINDING A MERIDIAN LINE

201 The knowledge of the direction of the meridian is useful for several purposes, but absolutely necessary for adjusting a transit instrument. The first step, and that the most difficult, is to find it nearly when this is done, it may easily be corrected by help of the transit instrument itself. Either of the two following methods, especially the second, will serve at once for finding it sufficiently near for most purposes, except for the transit instrument.

202 On an horizontal plane describe several concentric circles of a few inches in diameter. In the centre place a wire, a few inches long, at right angles to the horizontal plane Note in the forenoon the point where the shadow of the top of the wire just reaches any of the circles, and watch in the afternoon the point where the extremity of the shadow again reaches the same circle. The arch intercepted between these two points being bisected by a radius, the radius will be in the direction of the meridian; because the direction of the shadow is in the plane of the vertical circle passing through the sun, and the sun has equal azimuths at equal distances from noon, unless as far as the change of declination interferes.

This meridian may be transferred to any near place, by suspending a plumb line directly over the southern extremity of the line drawn as above, and noting when the shadow falls on that line · at this time another plumb line, suspended at the place where the meridian line is required, will, by its shadow, shew the meridian.

The imperfections of this method of finding a meridian line arise from the inexact termination of the shadow, and from the change of the sun's declination in the interval of the two observations The latter inconvenience is least in June and December near the solstices.

203 The other method is perhaps as simple and exact as can be expected without the assistance of a telescope, and is applicable, even with a transit instrument. Observe when the pole star above the pole, and ε Ursæ Majoris, called Alioth, are in the same vertical, a plane passing through these stars at that time is nearly in the plane of the meridian

The pole star and Alioth pass the meridian within about nine minutes of each other, the former being 1° 45′ above the pole, and the latter 33° below it Alioth passing the meridian below the pole, about nine minutes before the pole star passes above the pole, it follows that the vertical circle passing through the polar star, and approaching the meridian, will be met by the vertical circle passing through Alioth, receding from the meridian, and therefore Alioth and the pole star will be in the same vertical within less than nine minutes of time of the passage of the pole star: and as the pole star changes its azimuth very slowly, the vertical circle passing through these two stars must be nearly in the plane of the meridian.

204 The deviation of this vertical circle from the plane of the meridian may be easily computed: for in general the sine of the azimuth is to the sine of the hour angle at the pole, as the sine of the polar distance is to the sine of the zenith distance Now the mean R. Asc. on Jan 1, 1810,

Of α Polaris - = $0^h\ 51'\ 37''$

Of Alioth - - = 12 45 41

Hence when Alioth is on the meridian below the pole, the hour angle at the pole star ($= 8^m\ 56^s$ in time) $= 2°\ 14'$, and therefore the sine of the azimuth of the pole star when Alioth

passes $= \dfrac{\sin\ 2°\ 14'.\ \sin\ 1°\ 45'}{\sin.\ (\text{co lat.} - 1°\ 45')} =$ (in lat. 53° 23′,) sin. 7′ nearly.

This is the azimuth of the pole star, when Alioth is passing the meridian below the pole. When they are in the same vertical,

the common azimuth is somewhat less,[a] but the difference is so small, that it is scarcely worth notice in this approximation to the meridian, which serves without farther correction for most common purposes. The changes of the right ascension from aberration are not noticed, because the method is only given here for an approximation.

205. The following is a convenient way of *practising* this method. Suspend two plummets, A and B, (Fig. 29), to each end of a rod CD. Vessels of water should be used for steadying the plummets. A pivot fixed to the middle of the rod should be supported on a socket at E; so that the rod may turn steadily and freely. If Alioth and the pole star be observed in the plane of the plumb lines, that plane will be, in these latitudes, within about 7′ of the meridian. The eye will readily shew when they are nearly in the same vertical, and then the plumb lines, by turning the rod on its socket, may be easily made to pass through them, when exactly on the same vertical.[b]

206. Instead of the plumb lines, a transit telescope turning on an horizontal axis may be used. The deviation from the meridian of the telescope, so adjusted, may be found by observing the transits of a star to the south of the zenith, and of the pole star. The transit of the former will give the sidereal time

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[a] The correction of the azimuth is very easily computed, for the angular motion of the vertical of the pole star is to the angular motion of the vertical of Alioth, as $\frac{\text{sine polar dist. of pole star}}{\text{sine zenith dist.}} : \frac{\text{sine polar dist. of Alioth}}{\text{sin. zenith dist.}}$ = (in lat. 53° 23′) $\frac{\sin 1^\circ 15'}{\sin 34\ 52} : \frac{\sin 33^\circ}{\sin.\ 69\ 37} :: 1 : 11$ nearly. Therefore the azimuth of common vertical is to azimuth of pole star, when Alioth passeth, as 11 to 12 nearly; and therefore azimuth of common vertical $= \frac{11}{12} \times 7' = 6,4$ nearly.

[b] This method can only be used when the polar star passes the meridian above the pole, when it is dark, that is, from the end of August to the end of January. There are no other stars so convenient for this method, although Capella below the pole, and ε Ursæ Minoris above the pole, may serve.

nearly, and comparing the time so found with the sidereal time given by the polar star, the difference, which may be considered as entirely the error from the pole star, will give the deviation from the meridian: for the deviation in seconds of a degree is to error in seconds of a degree of sidereal time of transit of pole star, as the sine of the polar distance of the pole star to the sine of the zenith distance. The reason of considering the whole difference, as the error of the pole star, is, that when the deviation from the meridian is small, the error of sidereal time from a star, southward of the zenith is very small, compared to the error from the polar star. This is a very convenient method of approximating at pleasure to the meridian

207. The deviation from the meridian may also be found by comparing the times of continuance of a circumpolar star on the east and west sides of the meridian.[a]

A quadrant having an azimuth circle is very convenient for ascertaining the meridian, by observing equal altitudes on each side of the meridian, and then bisecting the arch of azimuth. If the sun be used, allowance must be made for the change of declination.

A good clock will serve instead of an azimuth circle, by observing equal altitudes of the sun or a star, half the interval of time corrected (if the sun is observed) will shew when the object was on the meridian, and thence the error of the clock will be ascertained, and so the time of the transit of any star may be computed, and the instrument adjusted at the time of that transit.

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[a] Professor Vince's Practical Astron. p. 82.

CHAPTER XII.

GEOCENTRIC AND HELIOCENTRIC PLACES OF PLANETS—NODES AND INCLINATIONS OF THEIR ORBITS—MEAN MOTIONS AND PERIODIC TIMES—DISCOVERIES OF KEPLER—ELLIPTICAL MOTIONS OF PLANETS.

208. The fixed stars, as has been noticed, appear in the same place with respect to the ecliptic from whatever part of the solar system they are seen, but not so the planets: their places as seen from the sun and earth are very different, and as their motions are performed about the sun, it is necessary to deduce from the observations made at the earth, the observations that would be made by a spectator at the sun. By this we arrive at the true knowledge of their motions, and discover that their orbits are neither circular, nor their motions entirely equable about the sun, although a uniform motion will, in some measure, solve the phænomena of their appearances.

It has before been shewn how the distances and periodic times of the planets are found, on the hypothesis of their orbits being circular, and their motions uniform; it remains to shew how the places of the nodes and inclinations of the orbits may be found nearly, before we proceed to more accurate investigations. For this, it is necessary to find from the geocentric longitude and latitude (computed, from the right ascension and declination observed,) and the distance of the planet from sun known nearly (art 97 and 101) the heliocentric latitude and longitude

209. Let S and E (Fig. 30) be the sun and earth, P the

planet, O its place, projected perpendicularly on the plane of the ecliptic, SA the direction of Aries, and EH parallel to SA. Then OEH and OEP are the geocentric longitude and latitude of the planet, and OSA and PSO are the heliocentric longitude and latitude. From the right ascension and declination observed, and the right ascension and declination of the sun, we can compute the planet's angular distance from the sun, or the angle SEP. For we have then the angle subtended at the pole between the sun and the planet, and the polar distance of each. Therefore in the triangle SEP, we know SP, SE, and the angle SEP; from thence we can deduce PE, and thence OE, because OE : PE : : cos. OEP (geocent. lat.) : rad. Hence in the triangle SOE, ES, OE and angle SEO (diff. long. of planet and sun) are known, and so we can compute OSE. Whence, because ESA = earth's longitude seen from sun = sun's longitude + 180° we obtain OSA the heliocentric longitude. Also because PS × sin. PSO = OP × rad. = EP × sin. OEP, we have sin. hel. lat. : sin. geo. lat. : : EP : PS, and thus the heliocentric latitude is known.

210. From two heliocentric longitudes and latitudes, the place of the node and inclination of the orbit may be found. Let AR and AS (Fig. 31) be two heliocentric longitudes, PR and QS the heliocentric latitudes, and N the ascending node. Then by spherical trigonometry $\frac{\text{sin. NR } (= \text{AR} - \text{AN})}{\text{tan. PR.}}$ = co-tan. PNR = $\frac{\text{sin. NS } (= \text{AS} - \text{AN})}{\text{tan. QS.}}$, or (by Theorem. for sine of the difference of two arches)

$$\frac{\text{sin. AR} \times \text{cos. AN} - \text{cos. AR} \times \text{sin. AN}}{\text{tan. PR.}} =$$

$$\frac{\text{sin. AS} \times \text{cos. AN} - \text{cos. AS} \times \text{sin. AN}}{\text{tan. QS.}}$$

hence is deduced tan. AN =

$$\frac{\sin AN}{\cos AN}=\frac{\sin. AR \times \tan QS - \sin AS \times \tan PR,}{\cos. AR \times \tan. QS - \cos. AS \times \tan PR.}$$

AN is the longitude of the ascending node, this being found, we have cot. PNR (inclin. of orbit) $= \frac{\sin (AS-AN)}{\tan. QS}$.

The best observations for ascertaining the place of the node, are those made when the planet is near its node on each side. the best, for ascertaining the inclination, are when the planet is farthest from the ecliptic

The above is applicable to finding accurately the place of the node and inclination of the orbit, provided we had the planet's distance from the sun, at each observation, accurate How these may be found, will appear farther on. Therefore thus far it has only been shewn, how the distances, periodic times, places of the nodes, and inclinations of the orbits, may be nearly found.

211. Among the most valuable observations for determining the elements of a planet's orbit, are those made when a superior planet is in or near opposition to the sun, for then the heliocentric and geocentric longitudes are the same And a number of oppositions being observed, the planet's motion in longitude, as would be observed from the sun, will be known. The inferior planets also, when in superior conjunction, have the same geocentric and heliocentric longitudes: when in inferior conjunction, they differ by 180°, but the inferior planets can seldom be observed in superior conjunction, or in inferior conjunction, except when they pass over, which they rarely do, the sun's disc Therefore we cannot so readily ascertain by simple observation, the motions of the inferior planets seen from the sun, as we can those of the superior

212. The principal element for determining the place of a planet, is the mean angular velocity about the sun, called the *mean motion.* The periodic time is considered as invariable; but neither the real motion in its orbit, nor its angular motion

about the sun are equable The periodic time, being constant, may be taken as the measure of its mean motion, or rather the mean angle described in any given time, as twenty-four hours (deduced by proportion, from 360° being described in the periodic time.)

If the planet's place in its orbit, as seen from the sun, at any time, be known, its place at any other time will be had within a few degrees or less, by adding its mean motion, in the interval, to the former place this is to be corrected according to the deviation of the true motion from the mean place.

To obtain accurately the periodic time of a planet Find the interval elapsed between two oppositions separated by a long interval, when the planet was nearly in the same part of the zodiac. From the periodic time known nearly, it may be found when the planet has the same heliocentric longitude, as at the first observation Hence the time of a complete number of revolutions will be known, and thence the time of one revolution. The greater the interval of time between the two oppositions, the more accurately the periodic time will be obtained, because the errors of observation will be divided among a great number of periods, therefore by using very ancient observations, much precision may be obtained.

213. The planet, Saturn, was observed in the year 228 B C March 2, (according to our reckoning of time) to be near the star γ Virginis, and at the same time was nearly in opposition to the sun The same planet was observed in opposition to the sun, and having nearly the same longitude, in Feb. 1714.

Whence it was found that 1943 common years, 118 days, 21 hours, and 15 minutes had elapsed while the planet made 66 revolutions It being readily discovered that the time of a revolution was 29½ years nearly, it was easily ascertained that exactly 66 revolutions had been completed in the above interval, and therefore it follows that $29^y\ 162^d\ 4^h\ 19^m$ is the time of

one revolution, which gives 2′ 0″, 58 for the mean motion in 24 hours. The above time of revolution is with respect to the equinoctial points, and, as the equinoctial points recede, the time of a complete revolution in the orbit will be had by finding the precession of the equinoxes in longitude in the above time of revolution, and thence computing, by proportion, the time the planet takes to go over the arch of longitude equal to the precession. In this way the time of a complete revolution is found to be $29^y\ 174^d\ 11^h\ 29^m$, this is called a *sidereal revolution*, because it is the time elapsed between two successive returns of the planet to the same fixed star, when seen from the sun The time of revolution with respect to the equinoxes, the same as the time of revolution with respect to the tropics, is called the *tropical revolution.*

In the same manner ancient observations have been used for the other planets Ptolemy has recorded several oppositions of Jupiter and Mars observed by him in the second century. From these Cassini computed, by the help of modern observations, the periodic times with much exactness Ancient observations have also been used for Venus. Mercury, before the invention of telescopes, could not be seen, when near either inferior or superior conjunction, and therefore for this planet modern observations only have been used. however its transits over the sun's disc have enabled us to obtain the periodic time with sufficient accuracy

214. The exact periodic time of the earth is readily found by a comparison of two distant equinoxes; the time of the equinox is known by observing the sun's declination before and after the equinox, and thence the time when the sun had no declination, may be computed by proportion. Comparisons of good observations, separated by a long interval, give the time of returning to the same equinox, or the length of a tropical year = $365^d\ 5^h\ 48^m\ 48^s$ and as the recession of the equinoctial points

is $50''\frac{1}{4}$ in a year, the sun will appear to move over this space in $20^m\ 23^s\frac{1}{2}$ Hence the periodic time of the earth or a sidereal year $= 365^d\ 6^h\ 9^m\ 11^s$.

215. The ancient observations of Jupiter and Saturn, compared with the modern ones, give the periodic time of the former greater, and that of Saturn less, than what are found by a comparison of the modern observations The cause of this is satisfactorily explained by the mutual attraction of these bodies to each other, and the quantity of variation has been computed by the help of physical astronomy.

The tropical year is less now than in the time of Hipparchus, according to the determination of Laplace, by about 10^s

216. The next inquiry is the deviation of a planet's motion from equable motion, for which the knowledge of the form of the orbit, and law of motion in that orbit, are necessary This brings us to the discoveries of Kepler, who first ascertained, from the observations of Tycho Brahe, that the planets move in ellipses about the sun, which is in one of the foci; that the law of the motion of each planet is such, that it describes about the sun equal areas in equal times, and that the squares of the periodic times are as the cubes of the greater axes of their orbits. Kepler, to whom we owe these important discoveries, was born in 1571, and distinguished himself early in the seventeenth century. Naturally possessed of a most ardent desire of fame, it was fortunate for the progress of astronomy that he applied himself to this science. He had the advantage of referring to the numerous and celebrated observations of Tycho Brahe; who having, with unwearied exertions, constructed instruments, far better than had ever been made, used them with equal assiduity in forming a connected series of most valuable observations. Tycho Brahe observed in the Island of Hume, near Copenhagen; from whence, in consequence of most unmerited treat-

ment, he was obliged to retire to Prague, whither Kepler, at his persuasion, came to reside

217 Kepler first applied himself to investigate the orbit of Mars,[a] the motions of which planet appeared more irregular than those of any other, except Mercury, which, from being seldom seen, had then been little attended to. He has left us the result of his inquiries, in his work, "De Motibus Stellæ Martis," which will always deserve to be studied as a record of industry and ingenuity. It will not be convenient to enter here into many particulars of his labours. One of the most remarkable is, his long adherence to the hypothesis, that the orbits of all the planets must be circular, because a circle is the most perfect figure The planet was supposed to move in a circle describing equal angles about a point (punctum æquans) at a certain distance from the sun In this he was sanctioned by all who had gone before him, and it was not till having in vain spent near five years in attempting to accommodate this hypothesis to the observations, that he could persuade himself to reject it. "Primus[b] meus error fuit viam planetæ perfectum esse circulum; tanto nocentior temporis fur, quanto erat ab authoritate omnium Philosophorum instructior et metaphysicæ in specie convenientior." He afterwards proceeded by a method in which all conjecture was laid aside. From the numerous observations of Tycho Brahe, that had been continued upwards of twenty years, he obtained many distances of Mars from the sun, and the angles at the sun contained by these distances, and at last discovered that the curve passing through the extremities of these distances was an ellipse, in this manner arriving at a conclusion, which he considered as fully repaying him for his trouble. His attempts, his repeated disappointments, all of which

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[a] This was merely accidental Vid Kepler De Motibus Stellæ Martis, p 53.

[b] De Motibus Stellæ Martis, cap 40, p. 102

he has ingenuously recorded ; his ready invention in surmounting difficulties ; his perseverance at last crowned with success , remain as highly useful examples to shew the value of genius and industry united His adherence to the circular hypothesis, which was principally supported by its antiquity, affords a useful illustration of the inconveniences that may arise from not taking experiment and observation for our guides , and by his ultimate success he may be said to have given an illustrious example of that method of philosophising, which a few years afterwards was so strenuously recommended by Lord Bacon

218 Kepler's method, by which he at last obtained the orbit of Mars, will serve as a plain example of the manner of finding the orbit of a planet, and therefore may be considered as proper for an elementary work, although the present advanced state of astronomy furnishes others more convenient, but not so simple

He considered the orbit of the earth as circular, the sun being at a small distance from the centre, which the observations of Tycho were not sufficiently accurate to contradict, the orbit of the earth deviating so little from a circle Thus he was enabled to ascertain with sufficient precision the relative distances of the earth from the sun at different times, and the angles described about the sun ; having discovered that the point of equable motion was not, as astronomers at that time supposed, in the centre of the circle, but in the continuation of the line joining the sun and centre, and equally distant from the centre as the sun [a]

Let T and E (Fig. 32) be two places of the earth, when Mars is in the same place of its orbit. (These times are known from knowing the periodic time of Mars) ; P Mars, and M its

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[a] The ancient astronomers had supposed this to be so with respect to the planets, but the hypothesis had been rejected by Copernicus It is only nearly true in the orbits that are of small eccentricity.

projection on the plane of the ecliptic, S the sun. The angles MTS and MES are known from observations: TS, SE, and angle TSE from knowing the orbit and motion of the earth. In the triangle TSE we can find STE and TES and TE From these angles we find MTE and MET, and thence by help of TE we compute MT. Knowing MT, TS, and the included angle, we find MS
MT : MS : : cot. PTM (geo lat.) : cot. PSM (hel. lat.) thus we obtain the heliocentric latitude Then cos PSM (hel. lat.) : rad. : : SM : PS.

219. By the numerous observations of Tycho Brahe, Kepler was enabled to verify the same distance from several pairs of observations, and also to find many different distances, and the angles at the sun contained by these distances. In this manner he also found the greatest and least distances. Supposing the orbit circular, he had from these the diameter of the circle, and he could deduce any other distance at pleasure, by which means he compared the distances computed on this hypothesis with the distances computed from observation, and found that the distances in the circle were always greater than the observed distances. Hence he was assured that the orbit was not circular, but oval. He was at last led to try an ellipse, having the sun in one of the foci: this he found to answer by a comparison of a great number of observations of Mars. He soon concluded the same true for all the planets, and soon ascertained that *each described equal areas in equal times round the sun.*

220 The last discovery of Kepler was, that the squares of the periodic times are as the cubes of the greater axes of the ellipses This discovery was made many years after the two former: he conceived there must be some relation between the motions of the respective planets, which led him to search for that relation, and the above law was the result, which seems to

have given him as much pleasure as any of his discoveries We now know that this remarkable proportion is a simple result, from the principle of universal attraction which pervades all bodies How great must have been the satisfaction of Newton, who first established the existence of universal gravity, and by the application of mathematical principles, shewed that the three famous discoveries of Kepler were necessary consequences of that universal property of bodies

221 It will not be convenient here to enter into a farther detail of the methods by which all the particulars of the elliptical motions of the planets have since been established. They may be found in the copious astronomical treatises of Lalande, Professor Vince, Delambre, and others.

The computations made from the elements of the elliptical motions, agree so precisely with observation, that not a shadow of doubt can remain, that the planetary motions are performed according to the above laws; and all that may be thought necessary here is to show briefly, how the geocentric place of a planet may be computed from the elements of its motion in an elliptic orbit about the sun, and so compared with the same given by observation.

222 When a planet is at its greatest and least distances from the sun, it is said to be in *Aphelion* and *Perihelion* The distance of the sun from the centre of the ellipse is called the *eccentricity of the orbit* If the angular motion of the planet about the sun were uniform, the angle described by the planet in any interval of time, after leaving Aphelion, might be found by simple proportion, from knowing the periodic time, in which it describes 360°: but as the angular motion is slower near Aphelion, and faster near Perihelion, to preserve the equable description of areas, the true place will be behind the mean place in going from Aphelion to Perihelion; and from Perihelion to Aphelion, the true place will be before the mean place

The angle at the sun contained between the true and mean place is called the *equation of the centre* The angle between the Aphelion and mean place is called the *mean anomaly,* and the angle between the true place and Aphelion, the *true anomaly.*

223. The tables give the mean place of the planet in its orbit at some given time, called the epoch, from thence the mean place at any other time may be found, either by the tables, or by proportion: if from this the place of the Aphelion be subtracted, the mean anomaly of the planet is obtained, and from thence the true place is to be found The numerous calculations, now requisite in astronomy, make it necessary that all the aid possible should be derived from tables. Accordingly the tables give the mean motion about the sun for years, days, hours, &c., the place of the Aphelion,[a] and the equation of the centre and distance from the sun, for different degrees of mean anomaly. Thus we obtain the true place of the planet as seen from the sun, and its distance from the sun. The difference between the place in its orbit and the place of the node gives its distance from the node; whence, from knowing the inclination, we can compute its angular distance on the ecliptic from its node, and also its angular distance from the ecliptic, and thus find its heliocentric longitude and latitude. Hence, knowing the earth's distance from the sun, and its place, as seen from the sun, we can compute, by the converse of the method in art 209, the geocentric latitude and longitude.

The best tables of the motions of the planets contain the corrections to be applied, on account of the mutual attraction of

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[a] The latest French tables reckon the anomaly from Perihelion, instead of Aphelion, as has been usual hitherto. This was done to make the mode of reckoning similar to that for comets, the motions of which are necessarily estimated from Perihelion, and the intention seems to be, that in future the anomaly of the planets should be computed in the same manner.

the bodies of the system, by which their motions are disturbed, and by which also their nodes and Aphelia slowly change their places.

224. The true place of the planet in the ellipse, or the true anomaly, may also be deduced from the mean in the following manner.

Let AN (Fig. 33) be the axis major of the orbit, S the sun, P the planet, NLIA a semicircle described on the axis major, and ACL = the mean anomaly. Draw IPD perpendicular to AN. The area ACL : area ALN :: mean anomaly : 180° :: time of describing AP : time of describing APN :: area[a] ASP : area APN :: (by prop. ellipse) area ASI : area ALN. Hence area ACL = area ASI. Therefore sector LCI = triangle SIC. But as LI is small, the space between the chord and arc is very small, and therefore the triangles LCI and SIC are nearly equal, and consequently CI and LS are nearly parallel, and the angle LSC = ICD nearly. The angle ICD is called the *eccentric anomaly*. The sum of the angles LSC and SLC = LCA = the mean anomaly, CL + SC = SA, and CL − SC = SN. Therefore (by trigonometry) Aphelion distance SA : Per. dist. SN :: tan ½ mean anomaly : tan ½ diff. angles LSC and SLC. Let this diff. = D. Then ½ mean anom. + ½ D = LSC = ICA (the eccentric anomaly) nearly. The eccentric anomaly so found may be easily farther corrected, at pleasure, in the following manner. Because the sector LCI = the triangle SCI, we have[b] LCI × CI² = sin. SCI × SC × CI or LCI =

$$\frac{\text{sin. ICD} \times \text{SC}}{\text{IC}}.$$

Consequently, if we use the eccentric anomaly just found, for ICD, the error of LCI will be less than that of the sine of ICD, in the proportion of SC to CI. Hence

[a] Art. 219.

[b] Not the degrees, &c. in LCI, but the measure of LCI to radius unity.

subtracting LCI so found, from the mean anomaly, a much nearer value of the eccentric anomaly will be had. Using this new eccentric anomaly as before, a still nearer value will be had, &c. Two corrections will nearly suffice for all the planets. This is one of the most obvious methods of correcting the eccentric anomaly found above, but not the best adapted to practice;[a] one much better may be derived from it.

The eccentric anomaly being found, the true anomaly may be easily deduced. For, from thence the angle ISC in the triangle SCI can be found, and tan. ISD : tan. PSD : : ID : PD : : axis major : axis minor. Therefore PSD is known.[b]

225. The problem for finding the true from the mean anomaly, or, which comes to the same, to divide the area of the semicircle, by a line drawn from a point in the diameter, in a given ratio, has long been celebrated, and known by the name of *Kepler's problem*; he first endeavoured to solve it in consequence of his discovery, that a planet describes equal areas in equal times, about the sun. No exact solution can be given; it must be done either by continued approximation, or by help of a series.

226. Astronomers were not long in adopting Kepler's discovery of the elliptical motions of the planets, but they long hesitated in adopting the equable description of areas, in consequence of the difficulty it involved of finding the true from the mean place. They instead thereof had recourse to such hypotheses for the law of motion, as would afford them easy rules for finding the true from the mean place, and at the same time would give the computed place nearly within the limits of the errors of observation. One of the most celebrated was that of

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[a] Vide Appendix.

[b] This, although obvious, is not best adapted for practice. For tan. $\frac{1}{2}$ eccent anom. : tan. $\frac{1}{2}$ true anom : : $\sqrt{\text{aph. dist}}$: $\sqrt{\text{per. dist.}}$ vide Appendix

Seth Ward,[a] known by the name of the *Simple Elliptic Hypothesis* its value was derived, not from its accuracy, but from the elegance of the analogy used. He supposed the motion equable about the focus in which the sun was not; and from thence it easily follows, that the Aphelion dist. : Perihelion dist :: tan $\frac{1}{2}$ mean anomaly tan $\frac{1}{2}$ true anom. The anomaly thus found, may sometimes differ in the orbit of the planet Mercury 33′ from the truth, and in that of Mars 7′[b] The laws of motion assigned by other authors differed less from the truth, but required more complex computations. As no satisfactory reason could be assigned for Kepler's law, any other law that appeared to shew with equal accuracy the motions of the planets about the sun, had an equal claim to attention. This occasioned the invention of several different hypotheses before the time of Sir Isaac Newton. but his discoveries having fully established the Keplerian law, they were soon laid aside.

The first approximation above given for the eccentric anomaly, may occasion an error of 5′ in the anomaly of Mercury, of 20″ in that of Mars, &c

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[a] It has generally gone by the name of Ward's Solution, yet he did not claim it as his own, but acknowledged himself indebted to Bouilliald for the hint that led him to it. The fact is, that Kepler himself was not ignorant of it as an approximation, but rejected it as not sufficiently accurate.

[b] Trans R. I Academy, vol. ix p 143, &c

227. The following table exhibits the elliptic elements of the orbits of the principal planets

	Merc	Ven	Earth	Mars	Jup	Sat.	Geor
Eccentricity of the orbit, the mean distance 1000	206	7	17	93	48	56	47
Places of the Aphelion seen from the sun	s 8 ° 11	s 10 ° 8½	s 9 ° 9½	s 5 ° 2	s 6 ° 11	s 8 ° 29	s 11 ° 17
Mean motion in 24 hours as seen from sun	° 4 ′ 5 ″ 3	° 1 ′ 36 ″ 8	′ 59 ″ 8	′ 31 ″ 27	′ 4 ″ 59	′ 2 ″ 5,6	′ 0 ″ 42,0
Greatest equation of centre or deviation from mean place	° 23 ′ 40	° 0 ′ 47	° 1 ′ 55	° 10 ′ 10	° 5 ′ 30	° 6 ′ 27	° 5 ′ 21

New planet Vesta

The eccentricity of the orbit - -			88
Place of Aphelion - - - -	2^s	9°	$20'$
Mean motion in 24 hours - - -		$16'$	$18''$

Juno

Eccentricity of the orbit - - -			254
Place of Aphelion - - - -	8^s	22°	$49'$
Mean motion in 24 hours - - -		$13'$	$35''$

Ceres.

The eccentricity of the orbit - -			79
Place of Aphelion - - - -	10^s	26°	$9'$
Mean motion in 24 hours - - -		$12'$	$50'',7$

Pallas.

Eccentricity of the orbit - - -			245
Place of Aphelion - - - -	10^s	1°	$7'$
Mean motion in 24 hours - - -		$12'$	$50'',9$

CHAPTER XIII.

ON THE MOTIONS OF THE MOON—SATELLITES—COMETS

228. The satellites also revolve in elliptic orbits round their respective primary planets, having the same law of periodic times, but considerable deviations from the equable description of areas take place, in consequence of the disturbing force of the sun on the satellites, and of the satellites on each other.

The moon being a solitary satellite, we cannot apply the law of the periodic time to it But its orbit is nearly an ellipse, and it nearly describes areas proportional to the times, the deviation from which arises from the disturbing force of the sun This ellipse, however, does not retain the same position; that is, its points of greatest and least distance, called its *apogee* and *perigee,* do not retain the same position, but move according to the order of the signs, completing a revolution in about nine years

The laws of the principal irregularities[a] of the moon were discovered long before the cause of them.

229. The greatest difference between the true and mean place of the moon, arising from its elliptic motion, or the greatest equation of the centre, is 6° 18′, and this is the most considerable deviation from its mean place. But besides the quick motion of the apogee, completing a revolution in nine years, the eccentricity of the ellipse is also variable. hence the motions of the moon appear so irregular, that it would have been almost

[a] The corrections for these irregularities (improperly so called) are styled *equations.*

impossible to have developed the elliptic motion from the phænomena; and therefore without a knowledge of the form of the planetary orbits, it is hardly to be supposed that an ellipse could have been applied for explaining the motions of the moon, although at first sight the superior advantage of being in the centre of the orbit might lead us to suppose that the laws of its motions would be more easily known.

230. The periodic time of the moon may be ascertained with great exactness from the comparison of ancient eclipses with modern observations. At an eclipse of the moon, the moon being in opposition to the sun, its place is known from the sun's place, which can, back to the remotest antiquity, be computed with precision. Three eclipses of the moon, observed at Babylon in the year 720 and 719 B. C. are the oldest observations recorded with sufficient exactness. By a comparison of these with modern observations, the periodic time of the moon is found to be $27^d\ 7^h\ 43^m\ 11^s\frac{1}{2}$, not differing a second from the result obtained by recent observations. Yet we cannot use those ancient observations for determining the mean motion at the present time; for by a comparison of the above-mentioned eclipses with eclipses observed by the Arabians in the 8th and 9th centuries, and of the latter with the modern observations, it is well ascertained that the motion of the moon is now accelerated. This was first discovered by Dr. Halley, and, since his time, has been perfectly established by more minute computations. For a considerable time the cause remained unexplained; till M. Laplace showed it to be a variation of a very long period, arising from the disturbance of the planets in changing the eccentricity of the earth's orbit. He has computed its quantity, which closely agrees with that deduced from observation. The moon's secular motion, the motion in a century, is now $7'\frac{1}{2}$ greater than it was at the time the above-mentioned eclipses were observed at Babylon.

231 The two principal corrections of the mean place of the moon, beside that of the equation of the centre, are called the *evection* and *variation*. The evection depends upon the change of the eccentricity of the moon's orbit, and sometimes amounts to 1° 20′. This was discovered by Ptolemy. The variation which was discovered by Tycho Brahe depends upon the angular distance of the moon from the sun, and amounts, when greatest, to 35′ The other corrections arise only to a few minutes. But the number of corrections or equations used at present in computing the longitude alone of the moon, are thirty-two, and in computing the latitude, twelve.

232. It was before mentioned, that the nodes of the lunar orbit move retrograde, completing a revolution in eighteen years and a half. This motion is not uniform. The inclination of the orbit remains nearly the same, but not exact. The motion of the apogee is subject to considerable irregularities: its true place sometimes differs 12½° from its mean place. This was known to the Arabian astronomers, but seems to have been first accurately stated by Horrox, whose extraordinary astronomical attainments will be afterwards noticed He shewed the law of its change, and gave a construction for determining its quantity, which was adopted by Newton.

233 On all these accounts the computation of the exact place of the moon from theory is very difficult, and the formation of proper tables is one of the greatest intricacies in this science.

No small degree of credit is due to the industry of those who, by observation alone, discovered the laws of the principal irregularities Ptolemy, by his observations and researches, determined the principal elements of the lunar motions with much exactness. Horrox, who adopted the discoveries of Kepler, formed, about the year 1640, a theory of the moon, founded partly on his own observations. From this theory, Flamstead, about the year 1670, computed tables, which he found gave the

place of the moon far more accurate than any other. Flamstead himself soon after furnished observations, by which Sir Isaac Newton was enabled to investigate, by the theory of gravity, the lunar irregularities, which he has given in his ever memorable work. Notwithstanding the field opened by the publication of the "Principia," and the known necessity of exact tables of the moon for the discovery of the longitude at sea, seventy years elapsed from the publication of that great work, before any tables were formed for the moon, which gave its place within one minute. Clairaut made, after Newton, the first considerable advances in the improvement of the lunar theory from the principles of gravitation Professor Mayer, of the university of Gottingen, first published tables, by which the moon's place might be computed to one minute. The ingenuity exhibited in his theory and tables, and the incredible labour exerted in their computation and verification, will always render his memory distinguished. He died in 1762, at the early age of thirty-nine, worn out by his great and incessant exertions. His widow received from the British parliament a reward of £3000. About the year 1780, Mr. Mason, under the direction of Dr. Maskelyne, to whom modern astronomy is so much indebted, improved, by considerable alterations and additions, the tables of Mayer. Till very lately these were the tables generally used. Improved tables have now been furnished by M. Burg of Vienna, which appear to give the place of the moon to less than twenty seconds. The improvements in these tables were founded entirely on the observations of Dr. Maskelyne, for which purpose 3600 places of the moon, observed at Greenwich in the space of about thirty years, were used.

The tables of M. Burg have been superseded by those of M. Burckhardt, which are now used in computing the *Nautical Almanac*, and *Conn. des Temps* They are probably more accurate, and certainly more convenient than those of M. Burg.

234. Eclipses of Jupiter's satellites furnish us with ready methods of finding the principal elements of their orbits. Their mean motions about the centre of Jupiter are deduced by observing, after a long interval, the time elapsed between two eclipses of the same satellite, when Jupiter is near opposition. In this manner the mean motion may be attained to with great accuracy The places of the nodes and the inclinations of their orbits, may be found by observing the different durations of the eclipses of the same satellite Their orbits are all inclined by angles less than 4° to the plane of Jupiter's orbit. The two first satellites move in orbits very nearly circular, as astronomers have not been able to detect any eccentricity. The third has a variable eccentricity The orbit of the fourth satellite is more eccentric The inclinations of their orbits, and the places of their nodes, are variable

The complete illustration of the motions of the satellites from gravity was, till about thirty years ago, a desideratum in astronomy The attraction of the satellites to each other principally occasions the difficulty M. Laplace has since fully developed their motions, and furnished Theorems, by which M Delambre has computed tables, which give the times of the eclipses with great exactness.

235 It is a very remarkable circumstance, that the mean longitude of the first satellite, together with twice that of the third, always exceeds three times the mean longitude of the second by 180°. From whence it follows, that the mean motion of the first, together with twice that of the third, is equal to three times the mean motion of the second M. Laplace supposes this was only nearly true with respect to the primitive motions, and that the mutual action of the satellites rendered the relation exact, as we find it. From the former relation it follows, that the three inner satellites can never be eclipsed at the same time.

The three inner satellites of Jupiter return to the same position, with respect to one another, in 437¾ days. Hence this is the period of the irregularities of the three first satellites arising from their mutual disturbance.

236 Little more is known of the satellites of Saturn than their periodic times and distances from Saturn, and that the planes of the orbits of the first six are nearly in the plane of the ring, while that of the seventh is considerably inclined to that plane.

ON THE ORBITS AND PERIODIC RETURNS OF COMETS.

237. When a comet appears, the observations to be made for ascertaining its orbit are of its declinations and right ascensions, from which the geocentric latitudes and longitudes are obtained These observations of right ascension and declination must be made either with an equatoreal instrument, or by measuring with a micrometer, the differences of the declination and right ascension of the comet, and a neighbouring fixed star. The observations ought to be made with the utmost care, as a small error may occasion a considerable one in the orbit. The orbits of the planets being elliptical, it would naturally occur to try whether the motions of the comets are not also in elliptical orbits. But here the difficulty is much greater than for the planets. For the latter we have observations in every part of their nearly circular orbits For the comets we have observations only in a small part of their orbits, which are very eccentric, and of which many make considerable angles with the ecliptic. Hence to determine the orbit of a comet, from such observations as we can make during its appearance, ranks among the most difficult problems in astronomy.

238 Before the time of Newton, astronomers either did not suppose their orbits elliptical, or despaired of being able to de-

termine them from observation Not long, however, before the publication of the "Principia," M. Doerfell, a German, found that the motion of the famous comet of 1680, might be nearly represented by a parabola, having the sun in its focus This comet appeared to approach the sun directly, and descend from it again in the same manner

When the action of gravity was subjected to calculations by the illustrious Newton, the theory of the motions of comets became perfectly understood, and it was concluded that their orbits in general were very eccentric ellipses But in computing an orbit from observations, all we are in general able to do, is to represent, with accuracy, the comet's motion while in the neighbourhood of the sun, and visible to us We can do this by supposing the orbit a parabola, and on that hypothesis, computing its elements, in which way we can determine its path with sufficient exactness to make the observed and computed places agree very nearly with each other.[a] It is seldom, indeed, that we can expect to compute the elliptic orbit from the few observations we are enabled to make We may, it is true, deduce many eccentric ellipses that will represent, with the same accuracy as the parabola, the apparent motion Were we to attempt to compute the exact ellipse, the necessary errors of observation would render our conclusions quite uncertain. Hence, in general, we have no knowledge of the axis, and consequently of the periodic time, but from former observations of the same comet.

239. There are five elements which we may consider as determining the identity of a comet: these are the Perihelion dis-

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[a] Sir Isaac Newton first gave the solution of this problem, which he calls "longe difficillimum." Different solutions have since been given by various authors The best seems to be that of Laplace, (Mecanique Celeste, tom 1, p 221)

tance, the place of the Perihelion, the place of the node, the inclination of its orbit, and its motion being direct or retrograde. If two comets, recorded in history, are found to agree in these circumstances, there can hardly be any doubt of their identity, and consequently we obtain the knowledge of its periodic time, and are enabled to point out the future appearances of the comet

240 Dr Halley found that the comet which he observed in 1682, agreed in these circumstances with that observed by Kepler in 1607, and with that observed by Apian in 1531, whence he foretold that it would return again in the latter end of 1758, remarking, that it would be retarded by the attraction of Jupiter Its motion was retrograde, and the elements of the orbit deduced by Dr. Halley from the observations of Apian in 1531, of Kepler in 1607, and of himself in 1682, also the elements deduced from the observations in 1759, were as follow:

Passage through Perihelion	Per dist Earth's dis unity	Place of Perihelion	Place of node	Inclination to ecliptic.
D H		s ° '	s ° '	° '
1531 Aug. 21 18	,567	10 1 39	1 19 30	17 51
1607 Oct 26 8	,587	10 2 16	1 20 21	17 2
1682 Sep 14 4	,583	10 2 52	1 21 16	17 58
1759 Mar 12 11	,585	10 3 8	1 23 45	17 40

This comet was retarded by the action of Jupiter, as Dr. Halley had foretold This retardation was more exactly computed by Clairaut, who also calculated the retardation by Saturn The result of his computation, published before the return of the comet, fixed April 15 for the time of the passage through Perihelion it happened on March 12. Dr. Halley's own computation appears also very exact, when it is considered that he did not allow for the retardation by Saturn

A comet was expected in 1789, because one observed in 1532 was supposed to be the same as one observed in 1661 Halley mentioned the probability of their being the same, but not with confidence, and the event has made it very doubtful whether they were the same.

241. An ingenious computation has been made by Laplace from the doctrine of chances, to shew the probability of two comets being the same, from a near agreement of their elements. It is unnecessary to detail at length the method here. It supposes that the number of different comets does not exceed one million, a limit probably sufficiently extensive. The chance that two of these, differing in their periodic times, agree in each of the five elements within certain limits, may be computed, by which it was found to be as 1200 : 1 that the comets of 1637 and 1682 were not different, and thus Halley was justly almost confident of its re-appearance in 1759. As it did appear then, we may expect, with a degree of probability approaching almost without limit to certainty, that it will re-appear in 1835 at the completion of its period. But with respect to the comet predicted for 1789, from the supposition that those of 1661 and 1532 were the same, the case is widely different. From the discrepancy of the elements of these comets, the probability that they were the same is only 3 to 2, and we cease to be surprised that we did not see one in 1789.

Comets that appeared in 1264 and 1556 are supposed to have been the same, whence this comet may again be expected in 1848.

242. A comet appeared in 1770, very remarkable from the result of the computations of Lexell, which indicated a period of only $5\frac{1}{2}$ years; it has not been observed since. There can be no doubt that the periodic time of the orbit, which it described in 1770, was justly determined; for M. Burckhardt has

since, with great care, re-computed the observations, and his result gives a periodic time of $5\frac{1}{2}$ years [a]

Lexell had remarked, that this comet, moving in the orbit he had investigated, must have been near Jupiter in 1767, and and would also be very near it again in 1779, from whence he concluded that the former approach changed the Perihelion distance of the orbit, by which the comet became visible to us, and that in consequence of the latter approach, the Perihelion distance was again increased, and so the comet again became invisible, even when near its Perihelion This explanation has been in a manner confirmed by the calculations of Burckhardt, from formulas of Laplace He has found, that before the approach of Jupiter, in 1767, the Perihelion distance might have been 5,08, and that after the approach in 1779, it may have become 3,33, the earth's distance being unity. With both these Perihelion distances the comet must have been invisible during its whole revolution The Perihelion distance in 1770 was 0,67.

243 This comet was also remarkable by having approached nearer the earth than any other comet that has been observed · and by that approach having enabled us to ascertain a limit of its mass or quantity of matter Laplace has computed, that if it had been equal to the earth, it would have shortened the length of our year by $\frac{1}{9}$ of a day Now it has been perfectly ascertained, by the computations of Delambre on the Greenwich observations of the sun, that the length of the year has not been changed in consequence of the approach of that comet by any perceptible quantity, and thence Laplace has concluded that the mass of that comet is less than $\frac{1}{5000}$ of the mass of the earth. The smallness of its mass is also shewn by its having

[a] Laplace Mécanique celeste Tom. I, p 223

traversed the orbits of the satellites of Jupiter without having occasioned any alteration in their motions From these and other circumstances, it seems probable that the masses of the comets are in general very inconsiderable; and therefore that astronomers need not be under apprehensions of having their tables deranged in consequence of the near approach of a comet, to the earth, or moon, or to any bodies of the system

CHAPTER XIV.

ON ECLIPSES OF THE SUN AND MOON—TRANSITS OF VENUS AND MERCURY OVER THE SUN'S DISC.

244. The eclipses of the sun and moon, of all the celestial phenomena, have most and longest engaged the attention of mankind. They are now in every respect less interesting than formerly; at first they were objects of superstition; next, before the improvements in instruments, they served for perfecting astronomical tables; and last of all, they assisted geography and navigation. Eclipses of the sun still continue to be of importance for geography, and in some measure for the verification of the tables.

ECLIPSES OF THE MOON.

245. An eclipse of the moon being caused by the passage of the moon through the conical shadow of the earth, the magnitude and duration of the eclipse depend upon the length of the moon's path in the shadow.

Let AB and TE (Fig. 31) be sections of the sun and earth by a plane, perpendicular to the plane of the ecliptic. Let ATV and BEV touch these sections externally, and BPG and AMN internally. Let these lines be conceived to revolve about the axis CKV; then TVE will form the conical shadow, from every point of which the light of the sun will be excluded. The spaces between GT and PV and between VE and MN will form the *penumbra*, from which the light of part of the sun will

be excluded, more of it from the parts near TV and EV than from those near PG and MN

The semi-angle of the cone (TVK) = sem. diam. sun (CTA)—horizontal parallax of the sun (TCK) The angle subtended by the semi-diameter of the section = SKV = TSK − KVT = horizontal parallax of the moon + horizontal parallax of the sun—semi-diameter of the sun

The angle of the cone being known, the height of the shadow may be computed For height of shadow : rad of earth : : rad : tan $\frac{1}{2}$ angle of cone, also the diameter of section of the shadow at the moon is known, for $\frac{1}{2}$ SO : dist. moon : tan. sem diam. of section of shad radius

The height of the shadow varies from 213 to 220 semi-diameters of the earth, and nearly varies inversely as the apparent diameter of the sun.

246 When the moon is entirely immersed in the shadow, the eclipse is total; when only part of it is involved, partial; and when it passes through the axis of the shadow, it is said to be central and total The breadth of the section of the shadow at the distance of the moon is about three diameters of the moon, therefore when the moon passes through the axis of the shadow, it may be entirely in the shadow for nearly two hours (art. 132.)

The angle SKV is, when greatest, about 46′ : therefore as the moon's latitude is sometimes above 5°, it is evident an eclipse of the moon can only take place when it is near its nodes.

247. The circumstances of an eclipse of the moon can be readily computed. The lat. of the moon at opposition, the time of opposition, the horizontal parallax of the moon, and diameters of the sun and moon are known from the tables. Let the circle OCK (Fig 35) represent the section of the shadow at the moon, EF the path of the centre of the moon, OC the eclip-

tic, and CL the latitude of the moon at opposition. In the right angled triangle CHL we know CL and HCL (= the inclination of the lunar orbit nearly). Hence we find HC and HL. HC never differs more than a few seconds from CL. From HC and CF (the sum of the semi-diameters of the section of the shadow and moon) we compute FH (= HE) and thence EL and LF. By the tables we can compute the angular velocity of the moon in its orbit relatively to the sun (or its opposite point C) at rest. Thence we can find the time of describing FL and LE, or the time from the beginning of the eclipse to opposition, and the time from opposition to the end. And as the time of opposition is known, the times of beginning and ending of the eclipse are known.

If the diameter *abHt* of the moon be divided into twelve equal parts, called digits; then, according to the number of these in *bt*, the eclipse is said to be of so many digits.

248. The greatest distance of the moon, at opposition, from its node, that an eclipse can happen, is above $11\frac{1}{2}°$, and is called its ecliptic limit. When the moon is nearest the earth, let CD (Fig. 36) represent the semidiameter of the shadow at the moon, and LD the semidiameter of the moon touching it; LN the apparent path of the moon, and N the place of the node. Then NC is the limit of the distance of the node from conjunction, at which an eclipse can happen.

Sin. angle N ($5° 17'$) : rad. :: sin. CL (sem. moon + sem. section = $63'$ when greatest) : sin. NC ($11\frac{1}{2}°$.)

249. If the moon's nodes were fixed, eclipses would always happen at the same time of the year, as we find the transits of Venus and Mercury do, and will continue to do for many ages: but as the nodes perform a revolution backward in about $18\frac{1}{2}$ years, the eclipses happen sooner every year by about nineteen days.

In 223 lunations, or 18 years, 10 days, 7 hours, and 43 mi-

nutes, or 18 years, 11 days, 7 hours, and 43 minutes, according as there are five or four leap years in the interim, the moon returns to the same position nearly with respect to the sun, lunar nodes, and apogee, and therefore the eclipses return nearly in the same circumstances. This period was called the Chaldean Saros, being used by the Chaldeans for foretelling eclipses.

250. From the refraction of the sun's light by the atmosphere of the earth, we are enabled to see the moon in a total eclipse, when it generally appears of a dusky red colour. The moon has, it is said, entirely disappeared in some eclipses.

The Penumbra makes it very difficult to observe accurately the commencement of a total eclipse of the moon; an error of above a minute of time may easily occur. Hence lunar eclipses now are of little value for finding geographical longitudes. The best method of observing an eclipse of the moon is by noting the time of the entrance of the different spots into the shadow, which may be considered as so many different observations.

ECLIPSES OF THE SUN.

251. From what has been said of the earth's shadow, it is easy to see that the angle of the moon's shadow is nearly equal to the apparent diameter of the sun. Hence we compute that the length of the conical shadow of the moon varies from 60¼ to 55¼ semidiameters of the earth. The moon's distance varies from 65 semidiameters to 56. Therefore sometimes when the moon is in conjunction with the sun, and near her node, the shadow of the moon reaches the earth, and involves a small portion in total darkness, and so occasions a total eclipse of the sun. The part of the earth involved in total darkness is always very small, it being so near the vertex of the cone; but the part involved in the Penumbra extends over a considerable portion of the hemisphere turned toward the sun: in these parts the sun appears partially eclipsed.

252 The length of the shadow being sometimes less than the moon's distance from the earth, no part of the earth will be involved in total darkness; but the inhabitants of those places near the axis of the cone will see an annular eclipse, that is, an annulus of the sun's disc will only be visible. Thus let THF, LU (Fig 37) be sections of the sun and moon. Produce the axis SV of the cone, to meet the earth in B. from B draw tangents to the moon, intersecting the sun in I and N The circle, of which IN is the diameter, will be invisible at B, and the annulus, of which IH is the breadth, will be visible.

It has been computed that a total eclipse of the sun can never last longer, at a given place, than $7^m\ 38^s$, nor be annular longer than $12^m\ 24^s$. The diameter of the greatest section of the shadow that can reach the earth is about 180 miles.

253 The general circumstances of a solar eclipse may be represented by a projection with considerable accuracy, and a map of its progress on the surface of the earth constructed. (Professor Vince's Astron. vol. 1.)

The phænomena of a solar eclipse at a given place may be well understood by considering the apparent diameters of the sun and moon on the concave surface, and their distances as affected by parallax. When the apparent diameter of the sun is greater than that of the moon, the eclipse cannot be total, but it may be annular.

From the tables we compute for the given place the time when the sun and moon are in conjunction, that is, have the same longitude From the horizontal parallax of the moon, given by the tables, at this time, we compute its effects[a] in latitude and longitude, by applying these to the latitude and longitude of the moon, computed from the tables, we get the apparent latitude and longitude, as seen on the concave surface; and

[a] Or rather the effects of the difference of the parallaxes of the sun and moon

knowing the longitude of the sun, we compute the apparent distance of their centres, from whence we can nearly conclude the time of the beginning and ending of the eclipse, especially if we compute by the tables the apparent horary motion of the moon in latitude and longitude at the time of the conjunction. About the conjectured time of beginning, compute two or three apparent longitudes and latitudes, and from thence the apparent distances of the centres, from which the time may be computed by proportion when the apparent distance of the centres is equal to the sum of the apparent semidiameters, that is, the beginning of the eclipse In like manner the end may be determined. The magnitude also of the eclipse at any time may be thus determined let SE (Fig. 38) be the computed apparent difference of longitude of the centres L and S, LE the computed apparent latitude of the moon. In the triangle LSE we have therefore LE and ES to find SL the distance of the centres. Hence mn (the breadth of the eclipsed part of the sun) $= Ln + Sm - SL$ is known.

254 The ecliptic limits of the sun (the greatest distance of the conjunction from the node when an eclipse of the sun can take place) may be found as follows let CN and NL (Fig 39) be the ecliptic and moon's path, and CN the distance, when greatest, of the conjunction from the node; as the angle N (the inclination of the orbit) may be considered as constant, when CN is greatest, CL, the true latitude of the moon, is greatest. The true latitude = apparent latitude ± parallax in latitude = (when an eclipse *barely* takes place) sum of the semidiameters + parallax in latitude. Therefore at the ecliptic limits the parallax in latitude is the greatest possible, that is, when it is equal to the horizontal parallax Hence CL = semidiameter moon + sem. diam. sun + hor par. moon.[a] Therefore CL (when

[a] It is scarcely necessary to mention the horizontal parallax of the sun in this investigation It should properly be the horizontal parallax of the moon—hor. par. sun.

greatest) =[a] 33′ + 61′ (= 1° 34′) nearly And because sin. NC = $\frac{\text{cot. N} \times \text{tan LC}}{\text{rad.}}$, we find NC = 17° 12′ nearly. An eclipse may happen within this limit; but if we take CL = 30′ + 54′ (the least diameters and least parallax) = 1° 24′ we find NC = 15° 19′ and an eclipse *must* happen within this limit.

255 There must be two eclipses, at least, of the sun every year, because the sun is above a month in moving through the solar ecliptic limits. But there may be no eclipse of the moon in the course of a year, because the sun is not a month in moving through the lunar ecliptic limits.

When a total and central eclipse of the moon happens, there may be solar eclipses at the new moon preceding and following, because, between new and full moon, the sun moves only about 15°, and therefore the preceding and following conjunctions will be at less distances from the node than the limit for eclipses of the sun As the same may take place at the opposite node, there may be six eclipses in a year. Also when the first eclipse happens early in January, another eclipse of the sun may take place near the end of the year, as the nodes retrograde nearly 20° in a year Hence there may be seven eclipses in one year, five of the sun, and two of the moon [b]

256. Thus more solar than lunar eclipses happen, but few solar are visible at a given place.

A total eclipse of the sun, April 22, 1715, was seen in most parts of the south of England A total eclipse of the sun had not been seen in London since the year 1140

The eclipse of 1715 was a very remarkable one, during the total darkness, which lasted in London 3m 23s, the planets Jupiter, Mercury, and Venus were seen; also the fixed stars Capella

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[a] Art 60, 83, and 132

[b] Or four of the sun and three of the moon —ED

and Aldebaran. Dr. Halley has given a very interesting account of this eclipse,[a] which is said by Maclaurin to be the best description of an eclipse that astronomical history affords. A particular account is also given in the Phil. Trans. by Maclaurin of an annular eclipse of the sun, observed in Scotland, Feb. 18, 1737. He remarks, that this phænomenon is so rare, that he could not meet with any particular description of an annular eclipse recorded. This eclipse was annular at Edinburgh during $5^m\ 48^s$.

257. The beginning and end of a solar eclipse can be observed with considerable exactness, and are of great use in determining the longitudes of places: but the computation is complex and tedious, from the necessary allowances to be made for parallax.

TRANSITS OF VENUS AND MERCURY

258. The planets Venus and Mercury are sometimes in inferior conjunction when near their nodes: they then appear as dark and well defined spots on the body of the sun. Mercury can only be seen by the assistance of a telescope, but Venus may be seen by the eye, defended with a smoked glass, or on the image of the sun formed in a dark room by an aperture in the window. Venus appears in a telescope, a well defined black spot, 57″ in diameter. The diameter of Mercury is only about 11″.

259. The transits of Mercury are much more frequent than those of Venus. This is merely accidental, arising from the proportion of the periodic time of Mercury to that of the Earth, being nearly expressed by several pairs of small whole numbers. If an inferior planet be observed in conjunction near its node

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[a] Phil. Trans. Vol. 29.

(or in a certain place of the Zodiac), it will be in conjunction in the same place of the Zodiac, after the planet and the earth have each completed a certain number of revolutions. Now it is easily computed from the periodic times of Mercury and the Earth, that nearly

7 per	of the Earth's rev.	=	29 per.	of Mercury's.
13	- of the Earth	=	54	- of Mercury.
33	- of the Earth	=	137	- of Mercury.

Therefore transits of Mercury, at the same node, may happen at intervals of 7, 13, 33, &c. years

8 per of the Earth's rev. = nearly 13 per. of the rev. of Venus

There are no intervening whole numbers till

235 per of the earth = nearly 382 per. of Venus.

Hence a transit of Venus, at the same node, may happen after an interval of 8 years. If it does not happen after an interval of 8 years, it cannot happen till after 235 years.

At present the ascending node of Venus, as seen from the sun, is in 2^s 14°, and the descending node in 8^s 14°. The earth, as seen from the sun, is in the former longitude in the beginning of December, and in the latter in the beginning of June Hence the transits of Venus will happen for many ages to come in December and June Those of Mercury will happen in May and November.

260. A transit of Mercury happened at the descending node in May, 1832, and the next will take place at that node in 1845. One happened in 1815 at the ascending node, another in 1822, and a transit will take place at that node in 1835.

In the years 1761 and 1769 there were transits of Venus, Venus being in her descending node. the next transit at that node will happen in the year 2004. But a transit was observed at the ascending node in the year 1639 by Horrox, who had previously computed it, from having corrected the tables of Venus

by his own observations, all other astronomers having been ignorant of its occurring. This transit will again happen at the end of 235 years from that time, or in the year 1874.

261. Horrox, who resided near Liverpool, when quite a youth, engaged in the study of astronomy with extraordinary enthusiasm and success. His having improved the tables of the motion of Venus so as to predict and observe this curious phenomenon, is one of the least of his astronomical performances.

He wrote an account of his observation in a dissertation, entitled, "Venus in sole visa," which, many years after his death, was published by Hevelius at Dantzic. This roused the attention of his countrymen to make inquiries respecting him, and to examine whether any of his manuscripts were remaining. A small part only of what were known to have existed, were found, and were published by Dr. Wallis about 20 years after his death.[a] Thus had not his manuscript "Venus in sole visa" accidentally fallen into the hands of Hevelius, there is reason to suppose, that in a few years, scarcely any trace of this extraordinary young man would have remained. The apparent neglect of his countrymen must be attributed to the civil wars, which almost immediately followed his death. He had no assistance in his labours, except from a friend, of the name of Crabtree, who lived at the distance of 20 miles. He also cultivated, with much ardour and ability, this science. Their correspondence is extant. Crabtree, informed by Horrox, ob-

[a] The account Dr. Wallis has given of the fate of Horrox's manuscripts is interesting. Some were brought to Ireland by his brother, who died here; these have never been found. Many were burned, during the civil wars of England, by some soldiers, who, searching for plunder, found them where they had been concealed. Some were used in composing a set of astronomical tables, called the *British Tables*, published in 1653. These were afterwards destroyed in the great fire at London, in 1666. The part that Dr. Wallis has published, was found in the ruins of a house at Manchester, in which his friend Crabtree had resided many years before.

served the transit at his own place of abode. Horrox died at the early age of 22, in the year 1641; and from what we see of his works that remain, it appears highly probable that, had his life been longer spared, his fame would have surpassed that of all his predecessors. His Theory of the Moon has been before mentioned (Art. 233.) He seems to have been the first astronomer who reduced the sun's parallax to nearly what it has since been determined. All astronomers before Kepler had made it more than two minutes: Kepler stated it at 59″; but Horrox, by a variety of ingenious arguments, evincing his superior knowledge in the science, shewed it highly improbable that it was more than 14″.[a] He also supposed that the disc of Venus, when seen on the sun, would not subtend a greater angle than 1′, whereas, according to Kepler, it would be 7′. Horrox, soon after he had entered on this science, was convinced by his own observations of the value of Kepler's discoveries.

262. The transits of the inferior planets afford the best observations for obtaining accurately the places of their nodes, and also the best observations for determining their mean motions.

The transits of Venus also afford us far the most accurate method of ascertaining the sun's distance from the earth, and therefore the magnitude of the whole system.

Dr. Halley first proposed this method of finding the sun's distance. He had observed, at the island of St. Helena, a transit of Mercury over the sun's disc, and thence had concluded that the total ingress and the beginning of the egress of Venus might be observed to 1^s of time; from whence, as he said, the sun's distance might be determined within $\frac{1}{500}$ of the whole distance. Experience afterward showed, that the times of total ingress and the beginning of egress could not be observed with certainty nearer than three or four seconds.

[a] Dr. Halley, above sixty years after, by arguments, not very dissimilar to those of Horrox, endeavoured to shew that it was not more than $12\frac{1}{2}$″.

263. To explain from whence the accuracy of this method arises, let us consider Venus and the sun as moving in the equator, and that observations of the total ingress are made at two places in the terrestrial equator: let AB (Fig. 40) be the equator, S and V discs of the sun and Venus, perpendicular to, and as seen from the equator. To a spectator at A the internal contact (or the total ingress) commences, when to a spectator at B, the edge of Venus is distant from the sun by the angle VBS. The difference then between the times of total ingress, as seen from B and A, is the time of describing VBS by the approach of the sun and Venus to each other, Venus being retrograde and the sun direct. Hence from this difference of times, and the rate at which Venus and the sun approach each other, we find VBS. And the sine of VBS : sine of VSB :: Venus's distance from the sun : Venus's distance from the earth. The relation of Venus and the earth's distance from the sun, as found by the method in art. 97, may be used. Therefore the angle VSB,[a] the angle subtended by the two places A and B at the sun is known, and consequently the angle the semidiameter of the earth subtends, will be found in a manner similar to that in the note of art. 58.

264. This simplification of the problem may serve for an illustration, and to point out its superior accuracy. But the actual computation of the problem is very complex, principally on account of the inclination of Venus's orbit to the ecliptic, and on account of the situations of the places of observation at a distance from the equator. The accuracy of the method consists in this: that the times of internal contact can be observed with great exactness, and thence the angle VBS computed, and therefore ASB.

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[a] For extreme accuracy the distance of the places A and B is to be diminished by the arch of the equator, described in the interval of the ingresses at each place.

At inferior conjunction, the sun and Venus approach each other at the rate of about 240″ in an hour, or 4″ in a minute Hence if the time of contact be erroneous at *each* place 4^s of time, the angle VSB *may be* erroneous $\frac{4 \times 8}{60} = \frac{8}{15}$ of a second, and therefore the limit of the error of ASB about $\frac{4}{15}$ of a second [a]

265 This method then in fact comes to the same as to find the angle at the sun, subtended by two distant places on the earth's surface, but this angle can be determined much more accurately by the times of ingress, than by the micrometer. On account of the difference of the apparent magnitudes of Venus and Mercury, the internal contact of the former can be determined much more accurately than of the latter.

This method requires the difference of longitude of the places to be accurately known, in order to compare the actual times of contact. The longitude of the Cape of Good Hope being well ascertained, observations of the transit of Venus in 1761, made there, were compared with many made in Europe, and the mean result gave the parallax = 8,47 seconds.

266. But it seemed more convenient not to depend on the knowledge of the difference of longitudes of two places. It appeared better to compare the differences of duration at two places, at one of which the duration was lengthened and at the other shortened. If we assume the parallax of the sun, which we know nearly, we can compute the difference of duration at any place from what it would have been, had it been observed

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[a] This comes to the same, as being able to observe a thread of light (the interval between the limbs of Venus and the sun, when the former has just entered upon the body of the sun) of only $\frac{4}{15}$ of a second in breadth Thus by the transit of Venus we can probably measure a smaller angle than by any other method

at the earth's centre.[a] Hence we can compare the difference of duration at two places, at one of which the duration is shortened and at the other lengthened Thus we shall have a double effect of the parallax, and we can compare the computed result with the difference observed From the error we can correct the horizontal parallax assumed

The transit of Venus in 1769 was observed at Wardhus in Lapland, and at the island of Otaheite in the South Sea.

Assuming the sun's parallax 8,83 seconds,

By computation the duration was lengthened at Wardhus	-	11^m 16^s,9
Diminished at Otaheite	-	12^m 10^s,0
Duration greater at Wardhus than at Otaheite	- - -	23^m 26^s,9
By observation	- - -	23^m 10^s,0

This shews the parallax is less than the parallax assumed, and to make the observed and computed difference of durations agree, the parallax must be taken $8''$,72 This last conclusion points out the accuracy of which the method is susceptible. difference of excess of duration of 17^s makes only a difference of $\frac{11}{100}$ of a second in the parallax

267. The observations of the transit of 1761 were not so well adapted for determining the sun's parallax as those of 1769 From the latter the parallax was ascertained with great exactness. The mean of the results seems to give $8''$,72 the sun's parallax at the mean distance, which probably is within $\frac{1}{10}$ of a second of the truth. The transit of 1769 occurring in the middle of summer, very many places of high northern latitude were well situate for observing it, but in all those the duration was affected in the same way

[a] See Dr. Maskelyne's Method and Computation, page 305 of Professor Vince's Astr vol 1

The duration is most lengthened when the commencement is near sunset, or when the sun is near the western horizon, and the end near sunrise or when the sun is near the eastern horizon. The duration of the transit in June, 1769, was about six hours That the commencement and end should take place under the circumstances above mentioned, it evidently required that the place of observation should have considerable north latitude Wardhus near the North Cape is in 70° 22′ N lat The commencement was there at 9^h 34^m in the evening, and end at 15^h 27^m

The duration would be most shortened when the commencement was near sunrise, and end near sunset, and the duration being only about six hours, this required that the days should be shorter than the nights, and therefore the place must be on the south side of the equator, and such that the commencement must be after sunrise and end before sunset. Consequently the choice of situations was much circumscribed.

Astronomers were therefore much at a loss for a proper place for observing this transit, when fortunately Otaheite was discovered. The situation of this island was as favourable as could be desired, and the British government, induced by a memorial from the Royal Society, ordered thither a ship with proper persons to make the observation In consequence of which, the first of the celebrated voyages of Cook took place The transit commenced at Otaheite about half past nine in the morning, and ended about half past three in the afternoon, and thus happened during the most favourable part of the day

CHAPTER XV.

THE VELOCITY OF LIGHT, AND ABERRATION OF THE FIXED STARS AND PLANETS—THE EQUATION OF TIME—DIALS

268 The velocity of light is the greatest velocity that has yet been ascertained Astronomy furnishes two methods of measuring it. Without the discoveries in astronomy, the velocity of light would have remained unknown. The eclipses of Jupiter's satellites, and the aberration of the fixed stars, shew us that the velocity of the reflected light of the sun, and the velocity of the direct light of the fixed stars, are equal

269 The elder Cassini suspected from observations of the eclipses of Jupiter's first satellite, that light was not instantaneous, but progressive. Roemer first fully established this fact, by a great variety of observations of the eclipses of the satellites of Jupiter.

Let the mean motion of a satellite be computed from two eclipses separated by a long interval, Jupiter being at each at its mean distance from the earth. Then an eclipse, when Jupiter is approaching conjunction, and therefore farther from the earth, happens later than is computed by the mean motion so determined.* When Jupiter is in opposition, it happens sooner than according to the mean motion so determined.

From a great variety of observations, it appears that the velocity of light is such, that, moving uniformly, it takes sixteen minutes to move over the diameter of the earth's orbit, or eight minutes in moving from the sun to us. This velocity is about

10,000 times greater than the velocity of the earth, which, as has been said, moves nineteen miles in a second (Art. 112.)

ON THE ABERRATION OF THE FIXED STARS AND PLANETS.

270 Another proof of the velocity of light is derived from the aberration of the fixed stars The fixed stars appear, by observations made with accurate instruments, to have a small annual motion, returning at the end of a year precisely to the same place. A star near the pole of the ecliptic appears to describe about the pole a small circle parallel to the ecliptic; the diameter of this circle is 40″ Stars in the ecliptic appear to describe small arcs of the ecliptic 40″ in length And all stars between the ecliptic and its poles appear annually to describe ellipses, the greater axes of which are parallel to the ecliptic, and equal to 40″. The axis minor is found by diminishing 40″ in the proportion of the sine of the star's latitude to radius. These phænomena cannot take place from the parallax of the annual orbit, because by it the latitude of a star would be greatest when in opposition to the sun, whereas then there is no aberration in latitude.

271 Dr. Bradley, who first discovered this apparent annual motion, when endeavouring to discover the parallax of γ draconis, also first explained the cause of it It arises from the velocity of the earth in its orbit, combined with the velocity of light [a]

272 The application of a few mathematical principles ena-

[a] Dr Bradley's own account of this phænomenon is very interesting, and is found in the Phil Trans vol 35 His observations were made with a zenith sector In the present state of astronomy, an instrument, whether a quadrant or transit, that will not readily shew the changes of the quantity of aberration, must be considered as a very inferior instrument.

bles us to explain and compute, with the greatest exactness, the laws of this phænomenon, which although not the most striking, is perhaps one of the most pleasing objects of astronomical contemplation. The apparent irregularities of the motions of the different stars, might, for a long time, have baffled the exertions of astronomers, had not the happy thought of applying the motion of light occurred to Bradley himself.

Let SA (Fig. 41) be the direction of light coming from a fixed star, and entering the telescope AD, carried in the direction DEF, by the motion of the earth If the direction of the telescope be the same as the direction of the rays of light, it is clear that no ray can come to an eye at D, as from the motion of the telescope with the spectator, they will be all lost against the interior of the tube. But if the tube be inclined in the position DB, so that BE : DE : : vel. of light : vel of the earth, then a ray SB parallel to SA entering the tube at B, will pass through the axis of the tube in motion, and be seen by the eye arrived with the telescope at E, while the light is passing from B to E. The ray of light will be always found in the axis of the telescope, carried by the motion of the earth, parallel to itself. The telescope being in the position EC, the star is judged to be in that direction, although it be actually in the direction EB. Hence BEC is the angle of aberration, and the aberration is always toward that part of the heavens, to which the earth is moving As BE is above 10,000 times greater than DE, it follows that the angle DBE must be very small, and therefore its equal BEC, the aberration must be very small. It is evident that DBE, and therefore BEC, is a maximum when BDE is a right angle, because sin. DBE : sin BDE : : DE : BE : : vel. earth : vel. light, a given ratio. Therefore when sin DBE is greatest, the sin. BDE is greatest, that is, when BDE is a right angle. Then vel. of light : vel of earth : sin. BDE (rad)

: sin of greatest aberr. and therefore sin of greatest aberr. =
$\frac{\text{rad} \times \text{vel of earth}}{\text{vel of light.}}$ = sine 20″ nearly

273 It may illustrate this matter, to consider the earth at rest, and the particles of light from the star having motions in two directions, viz the actual velocity of light in the direction BE, and another in a direction parallel and opposite to the earth or motion DE; by this compound motion, the particles of light would pass down the tube DB

To the naked eye the sensation must be the same, whether the light strikes the eye with a motion in the direction ED, or the eye strikes the light in the opposite direction; and therefore we may consider the light meeting the eye as coming in a direction compounded of two motions, that of light, and that of the earth, and therefore the same aberration takes place as in a telescope

274 The direction of the earth's motion is always toward the point of the ecliptic 90° behind the sun. Hence the stars all aberrate toward this point of the ecliptic, from which consideration the general phænomena of the aberration may be easily understood.

Also the phænomena of the aberration may be thus shown:

Conceive a plane passing through the star, parallel to the plane of the earth's orbit, and a line in this plane, parallel to the direction of the earth's motion, the length of which is to the star's distance, as the velocity of the earth to the velocity of light, the extremity of this line will be the place in which the star appears Now we may consider, without sensible error, the orbit of the earth as circular, and its velocity as uniform; therefore this imaginary line drawn from the star, parallel to the tangent to the earth's orbit, will be always of a constant length; and as the tangent in the course of a year completes a revolution, this imaginary line will also, in the course of a year, complete a revolution, and its extremity describe a circle about the

star. To a spectator on the earth, the star, in the course of a year, will appear to describe the circumference of this imaginary circle, the plane of which is parallel to the plane of the earth's orbit and he will orthographically project this circle on the concave surface, by which it will appear an ellipse. To find the axis major of this ellipse, we are to consider that the diameter of the circle of aberration, perpendicular to a circle of longitude passing through the star, will be projected into the axis major of the ellipse When the earth, seen from the sun, is in this circle of longitude, the line joining the star and earth will be at right angles to the direction of the earth's motion, and therefore the aberration will be then greatest, and equal to 20″ (Art. 272.) Hence the semiaxis major of the ellipse is 20″ The star's longitude is most increased when the star's and sun's longitudes differ by 180°, and most diminished when the longitude of the sun is the same as that of the star. When the sun's longitude exceeds that of the star by 90°, the radius of the circle of aberration is in the plane of the star's circle of longitude, and is diminished by projection on the concave surface, in proportion of the sine of the star's latitude to radius. The radius of the imaginary circle, thus diminished, becomes the semiaxis minor of the ellipse The star's latitude is most diminished when the sun's longitude exceeds that of the star, by 90°, and most increased when the sun's place is 90° behind the star

When the star is in the ecliptic, it is evident that the imaginary circle of aberration must be projected into a right line, or rather an arch of 40″ A star in the pole of the ecliptic appears to describe a circle 40″ in diameter, because the imaginary circle is not changed by projection. In practice it is necessary to compute the effects of aberration in right ascension and declination.[a]

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[a] Vid Appendix.

275 The aberration of a planet is somewhat different from that of a star; for if the planet's motion were equal and parallel to that of the earth, no aberration would take place From the small velocity of the moon about the earth, compared with the velocity of light, no sensible aberration takes place with regard to its velocity about the earth, and the moon and earth being carried together round the sun with nearly the same velocities, no aberration from thence occurs in the place of the moon.

The best method of finding the aberration of a planet or comet, is by first considering the effect of the earth's motion on the apparent place: this is the same as for a fixed star; and then the aberration arising from its own motion, this is readily computed, for the planet, supposing the earth at rest, appears in the place it was in at the emission of the light which reaches the eye, and therefore it is only necessary to compute the place of the planet for a time, so much earlier by the space of time that the light is coming from the planet to the earth.

276. The velocity of light determined by the eclipses of Jupiter's satellites has been considered as exactly the same as that determined by the aberration of the fixed stars [a]

As we are certain of the velocity of light by the eclipses of Jupiter's satellites, and also that the consequence of that velocity, and of the velocity of the earth, must be an aberration in

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[a] The maximum of aberration deduced from the velocity of light, as determined by the eclipses of Jupiter's satellites, appears to be 20″, 25 Bradley's observations appear to give the same quantity, but Bradley himself, on a revision of his observations, fixed it at 20″ But recent observations, made at the Observatory of Trinity College, Dublin, with the 8 feet circle, give it so great as 20″, 80 M. Bessel, from Dr. Bradley's Greenwich observations makes it 20″, 71. Lindenau, from observations of the pole star in R. Ascension, makes it 20″, 15 M. Struve, from observations in Right Ascension, makes it 20″, 00 It appears, therefore, highly probable, that it exceeds 20″, 25 By continuing the observations, it is hoped, greater certainty will be obtained in this important element

the fixed stars, we have, from the observation of the aberration, an independent proof of the motion of the earth

EQUATION OF TIME

277 The rotation of the earth on its axis is among the few perfectly equable motions known; the period of which, or 24 hours of sidereal time, might serve as a measure of duration, but this is not convenient for the purposes of civil life. For these, the period of a solar day, or the interval elapsed between two successive passages of the sun over the meridian, is a much more convenient measure of time. But this interval is variable, for it is greater than the time of the earth's rotation by a variable quantity This variable quantity is the time the hour circle passing through the sun takes to move over an arch equal to the increase of the sun's right ascension during a solar day Now the daily increase of the sun's right ascension is variable from two causes, viz, the inclination of the ecliptic to the equator, and the unequal apparent motion of the sun in longitude It is evident that the sun's increase of right ascension must be variable, on account of the obliquity of the ecliptic to the equator, because, when the sun is in Aries, its motion being oblique to the equator, the rate of increase of right ascension must then be less than the rate of increase of longitude, when at the tropics, its motion is parallel to the equator, and being nearer the pole of the equator than the pole of the ecliptic, its motion in right ascension must be then *greater* than its motion in longitude[a] Hence it is evident that the length of a solar day must

[a] It is not difficult to prove that the rate of increase of the sun's right ascension is, to the rate of increase of its longitude, as radius multiplied by the cosine of the obliquity of the ecliptic, is to the square of the cosine of the sun's declination The last term decreases from the equinox to the solstice, and therefore the first must increase.

be variable, and consequently that time, which is called *apparent solar time*, or apparent time measured by a solar day and parts of a solar day, must require a correction, which is called the *Equation of Time*. The perfection of the mechanism of a clock depends on the uniformity of its motion; therefore a clock intended to shew solar time, must be regulated according to mean[a] solar time, and the equation of time must be allowed in deducing apparent time from the time shewn by a clock. Apparent time is better adapted for civil purposes, mean time is necessary in computing the circumstances of the various celestial phænomena.

278. If the sun, instead of moving in the ecliptic, moved uniformly in the equator, the interval between two transits of the sun over the meridian would then be always the same, and would be an exact measure of time. Let us suppose then an imaginary sun moving uniformly over the equator in the same time in which the sun appears to move over the ecliptic, and having its right ascension, or distance from the beginning of Aries, equal to the mean longitude of the sun. The time measured by this imaginary sun so moving, is called *mean solar time* or *mean time*. The hour circle passing through the imaginary sun describes 360 degrees in 24 hours mean time, and that through the real sun the same in 24 hours apparent time, therefore each describes 15 degrees in an hour.[b]

279. The difference between mean and solar time, the *equation of time*, is evidently equal to the difference between the right ascension of the sun and the mean longitude of the sun, converted into time at the rate of 360° for 24^h, or 15° for 1 hour.

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[a] Art 199.

[b] The greatest difference between 24 hours mean time, and 24 hours apparent time is 30".

The sun's mean longitude is given by the solar tables, and thence the true longitude, by the latter, and the obliquity of the ecliptic, the right ascension may be computed, and then the difference[a] of mean longitude and right ascension, converted into time at the rate of 15° to an hour, is the equation Hence the computations for finding the right ascension of the sun, will also serve for finding the equation of time.

280. The changes of the quantity of the equation of time in different parts of the year, may be readily understood, for let VMPQEAONR (Fig. 42) represent the celestial equator extended into a right line, VJGQDLR the ecliptic, J the sun at the summer solstice, D at the winter solstice Take VG = 3ˢ 9°½, and G is the place of the sun when the earth is in Aphelion Take GQL = 180°, and L is the place of the sun when the earth is in Perihelion Let M, E, A, and N, be the places of the imaginary sun, when the sun is at G, Q, L, and R, or V, respectively. Then VM = VG, because at Aphelion the true and mean longitudes are the same[b], (Art. 222), therefore by spherical trigonometry M is between V and hour circle GP, that is, M is to the westward of the hour circle passing through the sun, and therefore mean time then precedes apparent time and because between G and L the true angular motion is less than the mean, (Art. 222), ME is greater than GQ = MQ, and therefore E is to the eastward of Q, consequently mean time then follows apparent time A is to the westward of the

[a] Accurately the equation of time is the difference between the sun's right ascension, and mean longitude reckoned on the equator from the true equinox, because the right ascension is computed from the true equinox By the sun's mean longitude, reckoned on the equator from the true equinox, is meant, the sun's mean longitude (always reckoned from the mean equinox) corrected for the equation of the equinoxes in right ascension.

[b] This is so, not taking into consideration the small effects of the lunar equation and equations for the disturbances of the planets

hour circle OL, because $QA = QL = 3^s\ 9^\circ\frac{1}{2}$, and therefore then mean time precedes apparent N is also to the westward of R, because from L to R the motion in longitude is greater than the mean motion, and therefore AN is less than $LR = AR$, and therefore then mean time precedes apparent. And, considering these circumstances, it will appear that between G and Q the equation vanishes, and also between Q and L, but not between L and R, but between V and G it twice vanishes. Thus mean and apparent time coincide four times in a year these times will be found to be about April 15, June 15, Aug 31, and December 24 The equation, it is easy to see, will be at its maximum, somewhere between Q and L; because when the sun is at Q, the mean sun will be behind it at E, and will become still more behind, because it moves faster in longitude than the true, and the effect of the increase of longitude of the sun is diminished by the obliquity of the ecliptic for some time after it has passed Q The maximum is $16^m\ 16^s$, and happens about the second of November.

A more particular consideration of the equation of time would be useless here. Indeed every thing of consequence may be considered as explained, when it is said to be equal to the difference, converted into time, between the sun's true right ascension and mean longitude, corrected for the equation of equinoxes in right ascension

281. It is to be observed, that the circumstances of the equation of time will change, with a change in the longitude of the earth's Aphelion, which moves forward from the equinox at the rate of $1'\ 2''$ in a year. The longitude at present, as seen from the sun, is $9^s\ 9^\circ\frac{1}{2}$. About 4000 years B. C (the supposed time of the creation) it coincided with the place of the earth at the vernal equinox.

The time shewn by a dial is apparent time, for it is the angle between the hour circle passing through the sun and the meridian, converted into time.

ON DIALLING

282. In a dial, the shadow of a straight line, by its intersection with a given plane, points out the apparent hour. The line by which the shadow is made, is called the style or gnomon. Let a meridian line be drawn on a horizontal plane, (art. 202, &c) and on this plane a gnomon or stile fixed, making an angle with the meridian line equal to the latitude of the place, and being also in the plane of the meridian This gnomon then will be in the direction of the celestial axis, (art. 39), the shadow therefore will always be in the plane of the hour circle in which the sun is, and because the sun is always in the same hour circle at the same distance from noon, whatever be its declination, it follows that the intersection of the shadow and horizontal plane is always the same at a given hour. Therefore these intersections of the shadow being marked, will always serve for pointing out the hour from noon These intersections are called hour lines of the dial, and a dial thus constructed is called an horizontal dial. The angles that these hour lines make with the meridian may be determined as follows

283 Let PO (Fig. 43) be the elevation of the pole, HP the hour circle 15° distant from the meridian, intersecting the horizon HO in H. Then HCO, C being the centre of the sphere, is equal to the angle between the hour line of one o'clock and the meridian on the dial, for CH is the horizontal intersection of the shadow of the axis PC at one o'clock.

By spherical trigonometry,

Rad. sin. PO (lat.) . . tan. HPO (15°) . tan. HO (HCO.)

Thus the angle which any hour line makes with the meridian, may be found, and a horizontal dial constructed.

If a vertical plane, facing the south, at right angles to the meridian, be used, the intersections of the shadow and this

plane, or the hour lines of the dial will be found, by computing the distances of the hour circles from the meridian on the prime vertical. A dial so constructed is called a vertical dial.

It is evident that the plane of the dial may make any given angle with the prime vertical, and the hour lines be readily computed by a spherical triangle When the plane of the dial faces the east or west, the stile is placed at a distance from, and parallel to its plane, because the plane of the dial is itself in the plane of the meridian.

CHAPTER XVI.

APPLICATION OF ASTRONOMY TO NAVIGATION—HADLEY'S SEXTANT—LATITUDE AT SEA—APPARENT TIME—VARIATION OF THE COMPASS—LONGITUDE AT SEA.

284. THE uses of astronomy in navigation are very great. It enables the seaman to determine by celestial observations his latitude and longitude, and thence discover his situation with an accuracy sufficient to direct him the course he ought to steer for his intended port, and to guard him against dangers from shoals and rocks. It also enables him to find the variation of his compass, and so affords him the means of sailing his proper course.

Almost all the astronomical observations made at sea, consist in measuring angles, and the difficulty of taking an angle at sea, on account of the unsteady motion of the ship, is sufficiently obvious. In taking an altitude, the plumb-line and spirit-level are entirely useless. In observing the angular distance of two objects, the unsteadiness of the ship makes it impossible to measure it by two telescopes, or by one telescope successively adjusted to each object.

285. These difficulties were soon seen when nautical astronomy began to be improved. Many attempts were made to invent a proper instrument. The ingenious Dr. Hooke proposed several methods. Many years afterwards Mr. Hadley proposed the instrument called Hadley's quadrant, now however usually called Hadley's sextant, for a reason that will be mentioned. A few years after Mr. Hadley's invention was communicated to

the world, a paper of Sir Isaac Newton's was found, describing an instrument nearly of the same construction. The principle of this invaluable instrument is, that in taking the angular distance of two objects, the image of one of them seen after two reflections, coincides with the other object seen directly, and this coincidence is in no wise affected by the unsteadiness of the ship. The operation by which the coincidence is made, measures the angular distance of the objects.

286. Let A and B (Fig. 44) be two celestial or very distant objects, HO, IN the sections of two plane mirrors, in the plane passing through the objects and eye. The mirrors are supposed to be perpendicular to this plane. Let a ray of light, AC, from the object A, incident on the mirror IN, be reflected in the direction CR, and so be incident on the mirror HO, from whence it is again reflected in the direction RE, coinciding with the direction of a ray, BR, from the other object, B. Then an eye any where in the direction of the line RE, will see the object A, coincident with the object B, if a portion of the mirror, immediately above the section HO be transparent. Thus we may make two distant objects appear to coincide by a proper position of the mirrors, viz., by inclining the mirrors at an angle equal to half the angular distance of the objects. For produce the sections of the mirrors, to meet in M, and produce AC to meet BRE in E. Then E = BRC − RCE = (by the principles of reflection) 2 HRC − 2 RCM = 2 M, or the angular distance of the objects equals twice the inclination of the reflectors. Hence if we move the reflector IN, so that both objects may appear to coincide, and can then measure the inclination of the reflectors, we shall obtain the angular distance of the objects. This principle is used in Hadley's sextant as follows:

287. ACB (Fig. 45) may represent the sextant. The angle ACB is 60°, but the arch AB extends a few degrees beyond each radius. A moveable radius, CV, called the index, re-

volves about the centre C, carrying a plane mirror, IN, perpendicular to the plane of the sextant, which mirror faces another mirror, H, also perpendicular to the plane of the sextant This latter mirror is fixed with its plane parallel to CA, the position of the mirror IN, when the radius CV passes through zero or (*o*) of the arch. The upper part of the mirror H is transparent, through which, by help of a telescope fixed at T, parallel to the plane of the sextant, the object S may be seen directly, while the image of M, seen by reflection, appears to touch it The angular distance of the objects M and S, is then, as has been shewn, = twice the inclination of the mirrors H and IN = (because H is parallel to CA) 2 VCA. Hence the degrees, minutes and seconds in VA, shewn by a vernier, attached to the extremity of the index, would give half the angular distance of the objects; but as the arch VA is only half the angular distance of the objects, for convenience each degree, &c is reckoned double; thus if VA be actually 42°, it is marked 84°, &c.

The mirror IC is called the *index glass*, and H the *horizon glass*, because in taking the altitude of the sun at sea, the horizon is seen, directly, through this glass.

In most sextants there is a provision for adjusting the plane of the horizon glass, parallel to the radius passing through zero of the arch, or rather parallel to the plane of the index glass, when the index is at zero of the arch. This is done by making an image coincide with its object seen directly, when the index passes through zero. Or the quantity of the error may be determined by measuring a small angle, for instance, the sun's diameter, on each side of zero of the arch. Half the difference is the error of the index, and it is most convenient to allow for this, as it cannot be corrected so exactly as its quantity can be ascertained.

For a more particular account of this instrument and its adjustments, see Professor Vince's Practical Astronomy

288 The best instruments, intended for taking the angular distance of the moon from the sun and stars, are made with great exactness. The radius of a sextant varies in length from five to fourteen inches. The usual length is about ten or twelve inches, and these admit of measuring an angle to 10″ or less, by help of the vernier Ordinary instruments are also made, merely for taking altitudes Plain sights are only used with these, and they are seldom adapted to take altitudes nearer than two or three minutes

As an altitude is never greater than 90°, it is evident, for an altitude, a greater arch than 45° is not required. The instruments, therefore, made only for taking altitudes, should properly be called *octants,* instead of quadrants, as they are sometimes named. The angular distance of the moon from a star is sometimes measured when 120°, for such distances an arch of 60° is necessary, and therefore the instruments intended for the longitude at sea are called *sextants*

In the octants, particularly, there is often a provision for measuring angles greater than 90°, by measuring the supplement to 180°, by what is called the back observation;[a] this is not often used.

289. The celebrated Mayer, whose lunar tables have been mentioned, recommended a complete circle for measuring the angular distance of the moon from the sun or stars by reflection, as in Hadley's instrument. Some of the advantages proposed, were similar to those of the astronomical circle over the astronomical quadrant; also by making the horizon glass moveable, the same angle could be repeated on different parts of the limb, and by repeating the angle many times, and taking a mean, the errors of division were almost entirely done away.

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[a] Professor Vince's Practical Astronomy.

Two causes may, perhaps, be assigned for this construction not having been at first adopted, the weight of the instrument rendered it inconvenient, and the superior skill of the London artists so constructed and divided sextants, that they seemed fully adequate to the purposes of the lunar method of finding the longitude in its early state. In its present state every minute source of accuracy is sought after, and it is now likely that reflecting circles will supersede sextants The French use an improvement of Mayer's circle by Borda In some reflecting circles made by Mr. Troughton of London, the advantage of the repeating principle is only in a small measure sought for. This is of less consequence, from the accuracy with which small circles may be divided by the machine invented by Mr. Ramsden; and otherwise Mr Troughton's circles seem more convenient than repeating circles for nautical purposes.

290. Let us proceed to the application of the sextant for finding the latitude, apparent time, variation of the compass, and longitude at sea

The latitude at sea is most readily and usually found by observing the meridian altitude of the sun At sea the horizon is generally well defined. The sextant being placed in a vertical position, the upper or lower limb of the sun, by moving the index, is brought down to the horizon seen directly The index shews the altitude; but it must be noted, that as the eye of the spectator is elevated above the level of the sea, the apparent altitude is to be diminished by the depression of the horizon, called the dip. The sun is known to be on the meridian when it ceases to rise higher, or when the index angle ceases to increase. An error of one or two minutes is of little consequence in finding the latitude at sea, as it makes only an error of one or two miles in the place of the ship. Oftentimes the horizon is not sufficiently defined to attain to great accuracy. A star can seldom be used, on account of the horizon not being

sufficiently visible, but the moon oftentimes may. The correct meridian altitude and the declination being known, the latitude is easily found, being always equal to the sum or difference of the zenith distance and declination

291 It often happens that it is cloudy at noon, and therefore an observation cannot be made · this sometimes is the case for several days together, when perhaps the sun is occasionally seen during that time. The latitude in such circumstances may be obtained by observing two altitudes of the sun, and noting the interval of time between, by a good watch · from these data and the declination the latitude may be found.

It may be mentioned, once for all, that it is here only intended to give a general account of the observations necessary for nautical purposes The particulars of the methods of computation are to be found in the different works on nautical astronomy, more especially in the work published by Dr. Maskelyne, entitled "Tables requisite to be used with the Nautical Almanac"

292 *The apparent time may be found at sea,* by observing the altitude of the sun. Then, knowing the latitude of the place and the sun's declination, we have the three sides of a spherical triangle, viz, the sun's zenith distance, the polar distance, and the co-latitude of the place, to find the hour-angle, which therefore may be had from one proportion. The hour-angle converted into time at the rate of 15° for one hour gives the apparent time from noon at the place of observation.

293 The latitude being known, the *variation of the compass* is easily found

Previously to the discovery of the polarity of the magnetic needle, navigators had no means of ascertaining their course upon losing sight of land, but by the sun and stars, particularly the polar star. They therefore seldom dared to venture far from land, knowing that a short continuance of cloudy weather

might occasion their destruction On the discovery of the compass, an end was put to this difficulty It must have been known at first that the needle did not point exactly north, but the deviation or *variation* was supposed every where the same So slow was the progress of navigation, that nearly two centuries elapsed from the time that the polarity of the magnet was well known in Europe, before it was discovered that in different places the variation was different Columbus, in his first voyage, seems to have been the first who observed it About a century later, the variation of the variation was discovered, that is, that the deviation from the north at a given place is variable The variation at London, two centuries ago, was 11° 15′ east, and is now 25° west.

294. On these accounts it is obvious, that the seaman must first ascertain the variation of the compass in the place in which he is, previously to his making use of it for his course this he practises by a very simple astronomical observation he notes, by the compass, the direction, called the bearing, of the sun when it rises or sets If the bearing is measured from the east or west, it is called the *amplitude* From the latitude of the place and the sun's declination, the azimuth at sun-rise or sunset may be computed by the solution of a right angled spherical triangle. For in the right angled triangle formed by the sun's polar distance, elevation of the pole and azimuth, cos lat. radius : : sin. dec. : cosin azimuth. The difference of the amplitude observed and computed gives the variation

Sometimes, the sun's azimuth and altitude are observed from the altitude, latitude, and declination, the azimuth may be computed, and thence the variation found or knowing the latitude, sun's declination and time of day, the azimuth may be computed, and then compared with the azimuth observed.

295. Places not far distant have nearly the same variation, except near the poles

It has been supposed that the variation of the needle, and latitude, would ascertain the position of a place, as well as its latitude, and longitude, and therefore that the variation of the needle would serve for finding the longitude But the variation cannot be obtained with sufficient accuracy to apply it to this purpose It seldom can be determined at sea, nearer than a degree.

296. The next subject to be explained, is the *method of finding the longitude at sea*

The difference of the apparent times at two places, found by the difference of the sun's angular distances from the meridian, at any instant, at each place, is the difference of longitude, the whole equator being considered as divided into twenty-four hours

297. If then we have the time of day at any place, the situation of which is known, and compare it with the time at the place in which we are, we obtain the difference of longitude. It is easy to find the time at the place we are in, (art. 292,) and therefore the finding its longitude is reduced to find the time of day at some given place, as at Greenwich, from whence we, in these islands, reckon our longitude

There are two methods of doing this: by time-keepers, or chronometers, as watches for this purpose are now usually called, and by making the motions of the celestial bodies serve instead of time-keepers.

298 It is evident, that did a watch or clock move continually at a uniform rate, it would afford us a ready means of finding the longitude: for if the chronometer, going mean time, were set to the time at Greenwich, it would continually point out the time at Greenwich, and therefore by comparing that time with the mean time at the ship, we should at once have the difference of longitude between Greenwich and the ship The apparent time at the ship can be found with all the accuracy

necessary, (art. 292,) and then applying the equation of time, the mean time will be obtained

299 It became therefore an object of great importance to construct a machine, the uniform motion of which might be depended on for a length of time

About the middle of the seventeenth century, Huygens and Hook made their celebrated improvements toward obtaining a regular movement in clocks and watches, the former by applying the pendulum to clocks, and the latter by applying a spiral spring to the balance of watches.

Huygens himself proposed the pendulum clock, for finding the longitude at sea, and quotes trials actually made; but it is obvious, on a variety of accounts, that a pendulum clock must be very unfit for a long voyage Watches also when made with the utmost care were found to be by much too irregular in their rates of going, to be depended on for a length of time.

Under these circumstances an act was passed in the reign of Queen Anne, in consequence of a petition from the merchants, for encouraging the discovery of a method of finding the longitude at sea within certain limits, for appointing a board of longitude, and for appropriating certain sums for encouraging attempts It was understood that the most desirable method, on account of its easy practice, would be by time-keepers. Mr. John Harrison early applied himself to the improvement of time-keepers, and during a long life was continually intent on that object. After many attempts which did his inventive genius the highest credit, and for which he received encouragement from the board of longitude, he at last completed a watch, which he considered perfect enough to entitle him to £20,000, the highest reward offered Accordingly in the year 1761, a trial was made by sending the watch to the West Indies, and he was considered as entitled to £10,000, and the remainder was to be granted to him upon strictly complying with the terms of

the act In the end, the whole, in consideration of his long and meritorious exertions, was granted to him

The act of Queen Anne only specified that to obtain the reward of £20,000, the error of longitude, in a voyage to the West Indies, should not exceed thirty miles This, in time, is about an error of two minutes Harrison's watch went within this limit: but it was soon found that the object of finding the longitude at sea, by time-keepers, was far from being attained The construction of Harrison's watch was extremely difficult. It seems that not more than one or two have ever been made on his principles. He may be considered as having led the way, and as having the credit of attempting the two principles of perfection, which have for many years past been introduced in the construction of chronometers

300. The two circumstances, by which chronometers differ from common watches, are, 1 The short time in which the main spring acts upon the balance This is accomplished by an escapement, called the detached escapement The action of the main spring is suspended during the greater part of the vibration of the balance, and therefore the isochronism of the balance spring is only slightly affected by the external impression of the main spring, through the intervention of the wheel work. 2dly. The contrivance for preventing the time of the vibration of the balance from being affected by heat or cold The balance, instead of being an entire circle, as in common watches, is composed of two arches (sometimes, but rarely, of three) to the end of each of which a small mass is attached the external part of the arch is brass, and the internal part steel these are soldered together, and from the different expansive powers of the two metals, by cold the arch becomes less curved, and by heat the contrary takes place. Thus the distance of the attached masses from the centre is always such as to preserve the isochronism. Chronometers well executed may be depended on to 1^s in a day.

These improvements in the construction of watches have been claimed by several artists, principally by the late Mr Arnold and Mr Earnshaw.[a] This is not the place to discuss, in any manner, their respective claims, or to enter into a comparison of the merits of the watches of different artists More has already been said than may be thought to belong to our subject, but the utility of chronometers, in their present state of perfection, is such as to have, in a manner, identified them with nautical astronomy. They are become extremely common, being furnished by several artists, at comparatively small prices, and are of most essential value on distant voyages. By them the longitude can often be found with great exactness, and by carrying on the reckoning, when astronomical observations necessary for finding the longitude cannot be made, they will serve to point out the longitude in the interim.

It is evident, that in long voyages, chronometers ought not to be trusted to, unless means of verifying them frequently offer; they are also subject to a variety of accidents that cannot be remedied at sea Hence the lunar method now to be described must be considered as much more valuable

301 Of all the celestial bodies, the moon is to us far the most convenient for the purpose of determining the longitude · its motion, as seen from the earth, being much quicker than that of the sun or any of the planets.

By the theory of the moon's motion, its place on the concave surface is known at any time; that is, knowing the time of the day at Greenwich, the place of the moon is known, and vice versa knowing the place of the moon, the time at Greenwich is known; so that if the lunar tables shew that the moon, seen from the centre of the earth, will be 10° from a certain fixed

[a] The merits of both these artists have been acknowledged by considerable grants from the Board of Longitude.

star, at six o'clock in the evening, at Greenwich, and we make an observation at any distant place, and find that the moon's distance from the star, reduced by computation to what it would be, seen from the centre of the earth, is 10°, we immediately conclude that it is 6 o'clock at Greenwich.

Thus the moon, with the brighter fixed stars near its path, may be considered as a chronometer, not made indeed by human hands, but perfect in its construction. It cannot, however, be easily used by us. The difficulty principally arises from the slowness of the apparent motion of the moon on the concave surface, and therefore great nicety is required in measuring the angular distance of the moon from the fixed star The intricacy of the lunar motions is also another source of difficulty.

But these inconveniences have now in a great measure been overcome by the improvements in instruments, and in the lunar theory; and navigators now use with much success this method

302. It is briefly as follows

The observer measures the moon's distance from the sun or a bright star in the zodiac by means of an Hadley's sextant or a reflecting circle This distance must be corrected for refraction, and reduced to the distance that would be observed from the centre of the earth, that is, corrected for parallax. The lunar tables are formed to give the place of the moon, as would be seen from the centre of the earth. For more readily computing the effects of parallax and refraction, another observer should, at the time of observing the distance, observe the heights of the moon and star. These altitudes need not to be observed with great accuracy

It being found by a reference to the tables at what time the moon was at this observed distance so corrected, the time at Greenwich is known.

To find the corrected distance, or to clear, as it is termed, the observed distance from the effects of parallax and refrac-

tion, let Z (Fig 46) be the zenith. The star is elevated in a vertical circle by refraction, and the moon is depressed by parallax and elevated by refraction also in a vertical circle Let RP be the apparent distance, R being the star and P the moon. In the vertical SZ take SR = the refraction of the star, and PM the difference between the moon's parallax and refraction, then SM will be the true distance

Let H = the app altitude of the sun or star
H' = the app. altitude of the moon
h = the true altitude of the sun or star
h' = the true altitude of the moon
A = the diff of apparent altitudes
a = the diff. of true altitudes.

Then by spherical trigonometry,

$$\frac{\cos A - \cos RP}{\cos H' \cos H} = \frac{\cos a - \cos SM}{\cos h \cos h'}$$ both these quantities being equal to the versed sine of the angle Z. Hence cos. SM = cos.

$$a - \frac{\cos h' \cos h}{\cos H' \cos H} (\cos A - \cos RP)$$

Different methods of shortening the computation of this formula, for the correct distance, are given in the works which expressly treat on the subject There are other methods by which the correction of the observed distance is obtained.[a]

303. The inconveniences of the lunar method of finding the longitude are,

1st. The great exactness requisite in observing the distance of the moon from the star or sun, as a small error in the distance makes a considerable error in the longitude. The moon moves at the rate of about a degree in two hours, or one minute of

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[a] Vid "Tables requisite to be used with the Nautical Almanac"
Mendoza's Treatise on Nautical Astronomy Mackay on the Longitude, Transactions Royal Irish Academy, Vol xi.

space in two minutes of time. Therefore, if we make an error of one minute in observing the distance, we make an error of two minutes in time, or 30 miles in longitude at the equator. A single observation with the best sextants may be liable to an error of more than half a minute: but the accuracy of the result may be much increased by a mean of several observations, taken to the east and west of the moon.

If the moon had moved round the earth in about three days, the longitude would have been as easily found as the latitude. The first satellite of Jupiter enables the inhabitants of that planet to find their longitudes with as great accuracy as can be desired.

2dly. The imperfection of the lunar tables has also long been considered as an obstacle in this method. The improved tables of Mason were frequently erroneous by nearly one minute, which occasioned an error of thirty miles. But there is reason to suppose that the error of the new tables of Burg and Burckhardt will rarely exceed 15″, which are only equivalent to seven miles and a half.

3dly. Another source of inconvenience is the length of the computation necessary in this method. Every thing possible was done by the late Dr Maskelyne for obviating this difficulty. He recommended the publication of the Nautical Almanac, which is now annually continued. In it the moon's distances from the sun and several zodiacal stars of the first and second magnitude, are given for every three hours. Such plain rules also, for reducing the observed distance to the true, have been laid down, more particularly in publications directed by him, that the computation is very short, and merely mechanical, so that it cannot be mistaken by a person tolerably versed in arithmetic.

304. The method above described is now universally practised in the service of the East India Company, and begins to

be held in much estimation in the navy The East India Company makes the knowledge of the practice of this method a necessary requisite in its officers

By it the longitude will be generally known to less than twenty miles, very often much nearer This, although less accurate than the latitude, is an invaluable acquisition to the seaman: it gives him sufficient notice of his approach towards dangerous situations, or enables him to make for his port without sailing into the parallel of latitude, and then, in the seaman's phrase, running down the port on the parallel, as was done before this method was practised Fifty years ago navigators did not attempt to find their longitude at sea, unless by their reckoning, which was hardly ever to be depended on. The difficulties they experienced are easily conceived

305. The present age must consider itself as principally indebted to the late Dr. Maskelyne, the Astronomer Royal, for the advantages which we derive from the lunar method of finding the longitude, and doubtless to him also posterity will acknowledge their great obligations He, by his own experience, on his voyage to St Helena in 1761, first satisfactorily shewed the practicability of this method. He strenuously recommended,[a] and then superintended the publication of the Nautical Almanac and of those tables, without the assistance of which, this method would have been of little value to the seaman To his observations is owing the present perfection of the lunar tables, and he unremittingly assisted and encouraged every attempt to forward the discovery of the longitude at sea, whether by this method or by time-keepers [b]

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[a] Vid. Dr Maskelyne's memorial, presented to the Commissioners of the Longitude, Feb 9, 1765, printed in the Appendix to Mayer's Tables

[b] The Theory of the lunar method is very old; indeed it is so obvious, that it could scarcely have been overlooked in the infancy of astronomy but the practice of it long seemed subject to insurmountable difficulties.

306. It has been supposed that the eclipses of Jupiter's satellites might be of great use in finding the longitude at sea. Experience, however, has shewn the contrary; it has been found impossible to manage a telescope on shipboard so as to observe the eclipses All attempts to remedy this difficulty have hitherto failed

CHAPTER XVII.

APPLICATION OF ASTRONOMY TO GEOGRAPHY—MEASUREMENTS OF DEGREES OF LATITUDE

307 ASTRONOMY furnishes several methods of finding latitudes and longitudes at land But the latter are found with much greater trouble, and less accuracy than the former The methods of finding the latitude of a place by observations made by the larger instruments, have been before mentioned, and it will here be only necessary to take notice of the use of Hadley's sextant for this purpose. By means of this portable instrument, the latitude may be found from observations of the sun's meridian altitude, with a degree of accuracy sufficient for many purposes of geography

308 At sea, the horizon is generally sufficiently defined to serve for measuring the sun's altitude, by Hadley's sextant; but at land, an artificial horizon is necessary, that is, we must make use of an horizontal reflecting surface, by which an image of the sun may be formed by reflection We measure, by the sextant, the angular distance between the upper or lower limb of the sun and its reflected image, which distance is twice the altitude of the limb, because the rays of light are so reflected that the angles of incidence and reflection are equal

There are various methods of forming this artificial horizon. Mercury and water afford the most convenient horizontal surfaces, when sheltered from the agitation of the air. For general use, perhaps, water ought to have the preference.

309 With respect to the longitudes of places at land, our

means of obtaining accuracy are much greater than at sea. We can repeat our observations at our leisure, and use such observations only as admit of the greatest precision. From the present state of Geography, as to the more known parts of the world, it cannot be much advanced by the lunar method of obtaining the longitude.

An occultation of a fixed star by the dark edge of the moon, observed at two places, the longitude of one of which is known, affords the greatest precision; because this phenomenon is instantaneous.

Eclipses of the sun rank next, but are not quite so accurate, because the beginning and end of an eclipse of the sun cannot be observed so exactly as the occultation of a star by the dark edge of the moon. The transits of the inferior planets also afford much accuracy.

The observations, however, which occur most frequently are the eclipses of the satellites of Jupiter. The first satellite passing more quickly into the shadow of Jupiter than the others, is best adapted for this purpose. By taking a mean of the results of the observations made on the first satellite, both in its immersions and emersions, great accuracy can be obtained.

310. By the assistance of a transit instrument, the longitude of a place can be had from observation of the difference of the times of the passages of the moon and a fixed star, compared with the difference observed at Greenwich or in some place of known longitude.

For the difference of the differences arises from the increase of the moon's right ascension in the interval of its passages over the respective meridians. From the rate of increase of the moon's right ascension is known the time corresponding to any given increase, hence the interval of time elapsed between the passages of the moon over the two meridians, and then the interval of sidereal time elapsed between the passages of the fixed

star over the two meridians, which is the difference of longitude

311. For particulars of the practice and computation of all the above methods, reference must be had to the larger astronomical works

The computations for occultations, for transits of the inferior planets, and for eclipses of the sun, are long and complex This arises from the effects of parallax, the phenomena not being observed at the *same* instant by each observer

The only difficulty, whether at sea or land, for finding the longitude, is to ascertain the time at a place where the longitude is known. This may be ascertained for near places as well by terrestrial signals, as by celestial observations. An eclipse of a satellite of Jupiter may be compared to a signal. An explosion or an instantaneous exhibition or extinguishment of a light being observed at two places, and the time noted exactly at each when it took place, the difference of longitudes will be had by simply taking the difference of the times In this manner considerable assistance has been afforded to Geography

312. But the mere knowledge of the latitudes and longitudes of places is not sufficient for the Geographer The exact figure and exact magnitude of the earth are also necessary in order to ascertain the exact distances of places, to describe and to plan the several countries.

On the hypothesis of the earth being a sphere, nothing more is necessary toward ascertaining its dimensions than *to measure the length of a degree of latitude* that is, to determine the length of an arch of a terrestrial meridian, the latitudes of the extremities of which differ by one degree The mode of ascertaining this is easily understood

The difference of latitude of two places in nearly the same meridian is to be ascertained by celestial observations. The distance, on the meridian, between these two places, is to be ob-

tained by terrestrial measurement A horizontal base line of a few miles in length, is to be measured in a convenient situation, and this base is then to be connected with the two places by forming a series of triangles, the angles of which are to be measured by a proper instrument, and then the distance of the two places computed by trigonometry

313 Let Q and T (Fig. 47) represent two places nearly in the same meridian QM. the line AC the base, the length of which is ascertained by actual measurement The angles of the triangles ACH, APH, NPH, PNQ, also of CHK and CTK are to be ascertained by an instrument adapted for taking angular distances Two angles of each triangle would be sufficient, as from thence the third angle is known but to verify the observations it is usual to observe all the angles of each triangle.

The base AC and the angles of the triangle ACH being known, the other sides AH and HC are had by computation, and thence the sides of the triangles APH, PHN, PNQ, CHK, and CTK

From T draw TMG perpendicular to the meridian QM, also let DQ, PE, and CF be perpendicular to QM, and PD, AE, AF, and CG parallel to the same

Now QM = DP + AE + AF + CG The sides PQ, PA, &c being known, PD, AE, &c will be had by the solution of right angled triangles, provided the angles DQP, EPA, &c are known These angles will be known if the angle PQM, or the angle that the direction of one of the stations P seen from Q makes with the meridian, be known. This angle may be obtained by different methods.

The sun being observed in the same vertical circle as the object P, the azimuth of the sun may be computed from the latitude of the place, the declination and distance in time of the sun from the meridian. thus the azimuth of P or the angle PQM will be had.

The pole star, when near its greatest elongation from the meridian changes its azimuth very slowly, and therefore is very convenient for ascertaining the direction of the object in respect to the meridian. The differences between the azimuth of the pole star, when at its greatest elongations east and west, and the azimuth of the object being obtained, half the sum or difference of these will be the azimuth of the object.

It is evident that when the inclination of PQ to the meridian is known, the inclinations of PA, AC, &c to the meridian and its parallels will also be known, because the inclinations of these lines to each other are known

The observations being made for ascertaining the length of QM, the difference of latitudes of the stations Q and P is to be observed with the utmost accuracy, by means of a zenith sector or other instrument affording sufficient exactness

For this purpose the zenith distance of a star near the zenith is to be observed at each place, and the sum or difference, according as the star is on a different, or on the same side of the zenith at each place, will give the difference of latitude. The changes in the apparent place of the star between the observations, arising from aberration, &c., must be taken into the account.

The length of the arc of the meridian, corresponding to a known difference of latitude, being thus found, the length of one degree will be had by a simple proportion

314. The minute particulars that must be attended to, in order to obtain the greatest accuracy, cannot be enumerated here. They are to be met with in the several accounts of the modern measurements

If the instrument, used in measuring the angles, give the angular distance and not the horizontal angular distance between the objects, the elevations or depressions must be also observed, that the horizontal angles may be computed

The triangles formed are not plane triangles, but spherical triangles not differing much from plane The sum of the three angles of each, is therefore somewhat more than 180°; but this excess is easily computed, and therefore the sum of the three angles may be still used for verification.

The computations of spherical triangles being more difficult than of plane triangles, mathematicians have devised ingenious methods to reduce the computation of these spherical to plane triangles, being assisted by the small difference between them and plane triangles.

315 The results of different measurements have shown that the degrees towards the poles are longer than those nearer the equator; and therefore that the earth is not exactly a sphere. This will be better understood by a short account of the principal steps by which we have arrived at our present knowledge of the form and dimensions of the earth.

316 The first modern measurement distinguished by a tolerable degree of accuracy is that of Norwood in 1635 He ascertained the difference of the latitudes of London and York, and then measured their distance, allowing for the turnings of the roads and for the ascents and descents. From which he deduced the length of a degree = 122,399 English yards According to the latest determinations it should have been = 121,660 yards

At this time no circumstances were known, which could tend to a knowledge of the exact figure of the earth

In the year 1671 it was discovered, by a comparison of the times of vibrations of the pendulums at Cayenne and Paris, that the weights of bodies were less near the equator than at Paris From whence Huygens considered it probable that the form of the earth was not spherical, but that it was a figure formed by the revolution of an ellipse about the lesser axis Sir Isaac Newton, arguing from juster principles than those of

Huygens, was also led to the same conclusion, and actually computed the ratio of the equatoreal and polar diameters, on the hypothesis of the earth having been at first an homogeneous fluid, revolving on its axis The ratio of the equatoreal to the polar diameter he found to be 230 . 229 At this time, 1686, no evidence from actual measurement existed, but Newton lived till it was ascertained by observation, that the ratio of the polar and equatoreal diameters of Jupiter was *nearly* such as his theory gave on the Hypothesis of an uniform density. He also lived till the results of actual measurements made in France appeared entirely inconsistent with the form which he had assigned. Subsequent measurements, made soon after Newton's death, fully established that the equatoreal exceeded the polar diameter.

317. Picard in 1670 measured an arc of the meridian, commencing near Paris and extending northward, and found, in latitude 49½°, a degree = 121,627 yards, differing only by about 35 yards, from what is now considered as the most exact length This accuracy seems to have been accidental, and obtained by a compensation of errors

A few years afterward, by order of the French King, Cassini, assisted by several other astronomers, undertook the measure of the whole arc of the meridian extending through France from Dunkirk to Collioure. This work was finished in 1718 Among the results obtained, it was found, that in latitude 46° a degree of the meridian = 121,708 yards, and in latitude 50° = 121,413.

Thus the degrees appeared to diminish as the latitude increased, instead of the contrary. For it is evident that if the curvature of the earth diminish as we recede from the equator toward the poles, the degrees of latitude ought to increase, because the less the curvature, the greater space must be gone over to change the elevation of the pole by one degree. This result therefore appeared to contradict Newton's conclusion, that the

earth was nearly an oblate spheroid, that is a solid, formed by the revolution of an ellipse about its lesser axis To support Newton's conclusion, it was objected that these degrees were so near each other, that the errors of observation and measurement might greatly exceed the difference of degrees that would come out from computation by Newton's figure But this mode of getting over the difficulty was not satisfactory. It was still contended by some of the French Academicians that the polar diameter of the earth was greater than the equatoreal

To remove all doubt, it was proposed that two degrees should be measured, one, as near to the equator, and the other as far northward, as conveniently could be done.

Accordingly in 1736, a company of French and Spanish astronomers went to Peru, to measure an arc near the equator, and a company of French and Swedish astronomers undertook to go to Lapland and measure an arc near the Arctic circle

The interesting particulars of their labours and difficulties have been minutely described by themselves, and their exertions for attaining the utmost accuracy cannot be sufficiently admired.

From a comparison of the measurements in Peru and in France, the equatoreal diameter[a] appeared to exceed the polar by about $\frac{1}{304}$ part of the whole

From a comparison of the measurements in Lapland and in France, the excess appeared to be $\frac{1}{210}$.

Thus the principal point was settled, that the earth was

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[a] If the density of the earth were uniform, and if the earth had been originally in a fluid state, its form would be accurately that of a spheroid, generated by the revolution of an ellipse about its minor axis The proportion of its diameters would then be readily investigated from a comparison of the lengths of two degrees of latitude (Vince's Astronomy, Vol ii p 98) As, however, the exact form of the earth is not known, the investigation of the proportion of the diameters from the comparison of two degrees of latitude is only to be considered as a near approximation.

flatter towards the poles, but the quantity of that flatness seemed by no means ascertained The measures in Lapland and in Peru seemed quite discordant. But from several circumstances, greater confidence was placed in the measure in Peru than in Lapland, although the latter seemed executed with all due care.

318. Arcs of the meridian have since been measured in several countries; but till very lately, no satisfactory conclusion was drawn respecting the degree of ellipticity in the earth, and even now greater exactness is desired

In the year 1787, it was determined to connect the observatories of Greenwich and Paris by a series of triangles, and to compare the differences of longitudes and latitudes, ascertained by astronomical observations, with those ascertained by actual measurement The late Major General Roy conducted the British measurement. The British Triangles were connected with those of the French, by observations made across the straits of Dover. In this manner assuming the latitudes of the respective observatories, as had been previously ascertained, it was found that in latitude 50° 10′ a degree of the meridian was 121,686 yards.

The measurement in England, which was begun with a reference only to the relative situations of the observatories of Greenwich and Paris, was extended to a survey of the whole kingdom. This, General Roy having died, was conducted by Colonel Mudge, with great skill and assiduity. In the course of his survey, in the year 1801, he measured an arc of the meridian, between Dunnose in the Isle of Wight and Clifton in Yorkshire The difference of latitude (nearly three degrees) was ascertained by an excellent zenith sector, made for the occasion.

From this measurement it resulted, that the length of a degree in latitude 52° 2′ = 121,640 yards

319 An arc of the meridian of nearly 10° in length has been measured in India, between a station near Cape Comorin, in lat. 8° 9′, and a station in the Nizam's dominions in latitude 18° 3′ This has been achieved by the exertions of Major Lambton continued during several years He was furnished with excellent instruments, similar to those used by Colonel Mudge The result of Major Lambton's measurement gives 120,975 yards for the length of the degree in latitude 13°. 3′ N.

A comparison of the degrees ascertained by Colonel Mudge and Major Lambton, gives the excess of the equatoreal above the polar diameter $= \frac{1}{313}$

320 At the time the English measurement was going on, the French astronomers Mechain and Delambre engaged in measuring the arc of the meridian from Dunkirk to Barcelona, which places are nearly under the same meridian, and differ in latitude by about $9\frac{1}{2}$°. Their operations commenced in 1792, and after struggling with the greatest difficulties arising from the unhappy situation of their country, they succeeded in accomplishing the objects of their labours From this measurement, compared with the measurement near the equator in 1736, &c, they deduced the excess of the equatoreal above the polar diameter $= \frac{1}{331}$.

321 In the year 1802, M Swanberg and other Swedish astronomers undertook to repeat the operations of the French Academicians, which they had performed near Tornea in Lapland in 1736 This was an object of considerable importance, on account of the different results deduced from the comparisons made with the measurements in France and Peru

M. Swanberg has given a most able detail of this operation and of the computations. The result which he deduces from a comparison with the new measurement in France, is an excess of the equatoreal above the polar diameter $= \frac{1}{307}$.

A comparison of the measurement of Major Lambton and of his own, gives the excess, the same, viz. $\frac{1}{307}$.

Other comparisons incline him to fix the most probable excess at $\frac{1}{323}$

The discordance of the degree measured in Lapland in 1736 and 1802, led to an examination of the source of the difference; and it appeared that the French Academicians had erred ten or eleven seconds in the latitude of one of their stations. All their other measurements were verified This error was sufficient to account for the difference of results

322 After all that has been done, much uncertainty remains as to the true figure of the earth: several measurements of degrees of longitude, compared with the degrees of latitude, give a much greater difference of diameters: however the measurement of a degree of longitude cannot be so accurate as that of a degree of latitude, on account of the difficulty of ascertaining the difference of longitudes of the extremities

323. The operation of measuring a degree of latitude consists in ascertaining the length of the arc of the meridian, and in ascertaining the difference of latitudes of the extremities. The latter part is not susceptible of near so great accuracy as the former A second in latitude answers to about 33 yards, and the difference of latitude cannot be probably ascertained nearer than two seconds, supposing no cause of irregularity to affect the plumb line. But there is sufficient proof that the plumb line is sometimes displaced several seconds by the attraction of mountains or of different strata Colonel Mudge and the French astronomers experienced this, in a considerable degree

The terrestrial measurements are susceptible of great accuracy It is usual to measure a base of verification, as far distant from the first base as can conveniently be done, and then compare this base with its length deduced by computation, from the first base and the angles measured: this was done by the French astronomers in their late survey The length of the

base of verification measured was upward of 7 miles, and at the distance of above 400 miles from the former base, and yet it did not differ by 12 inches from the length inferred by computations

324 The instruments used in the English measurement, and in that by Major Lambton, were a steel chain, an instrument for taking horizontal angles, the circles of which were 3 feet in diameter, and a zenith sector Mr. Ramsden exerted his great talents in making the construction of these instruments as perfect as possible

The first base in the English measurement was above five miles in length, and was measured in 1787 by glass rods · it was again measured in 1791 by the steel chain, and the two measurements differed only by about 3 inches

The instrument for taking the angles, sometimes called Ramsden's Theodolite, besides the accuracy it afforded, gave at once the horizontal angles, in which it had a great superiority over the instruments by which the angular distances between the stations were taken, and which afterwards required to be reduced by computation to the horizontal angles

325. In the recent measurements in France and Lapland, a repeating circle, of which the radius was only a few inches, was used for taking the angles and making the observations for the difference of latitudes of the extremities of the arcs. However inadequate at first sight such an instrument may appear to obtain conclusions in which extreme accuracy is required, it must be allowed that it fully answered the purposes for which it was intended. The length of the computation was much increased, as the angles observed were to be reduced to the horizon, and other reductions made . but these inconveniences seem much more than compensated by the portableness of the instrument.

The French base was measured by rods of platina the Swedish by rods of iron . the requisite allowance was made for the changes of temperature during the operations.

326. The result of the measurement in France has been used to ascertain a standard of measure. The length of a quadrant of the meridian was computed and found to be 5,130,740 toises or 10,936,578 English yards. This was divided into ten million parts, and one part, which was called a *metre*, was made the unit of measure. All other French measures are deduced decimally from this. The French metre then is 1,0936578 yards, or 39,37 inches nearly.

Computing from the length of the degree in latitude 45° the mean diameter of the earth comes out 7912 English miles nearly, and adopting the fraction $\frac{1}{320}$, the equatoreal diameter will exceed the polar by about 25 miles.[a]

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[a] A relation of the measurement in Lapland in 1736, was published by Maupertuis, and also by the Abbé Outhier, which is more minute than that of Maupertuis, (vid. Conn. des Temp. 1808.) Separate accounts of the measurements in Peru, were published by Ulloa, Bouguer, and Condamine.

A very particular account of the measurement in France was published by Cassini in 1744.

The particulars of the recent measurement in France have been published by Delambre, and of that in Lapland by Swanberg; (vide Conn. des Temps, 1808.)

An account of the measurement by General Roy, will be found in the Phil. Trans. for 1787 and 1790. Of that by Col. Mudge in the Phil. Trans. for 1803.

The latest account of Major Lambton's measurement is given in the Phil. Trans. 1818, p. 2.

An interesting account of the different measurements is also given under the article "Degree" in Rees's Cyclopædia.

CHAPTER XVIII.

ON THE CALENDAR

327 AMONG the different divisions of time, the *civil year* is one of the most important The *solar year*, or the interval elapsed between two successive returns of the sun to the same equinox, includes all the varieties of seasons.

The civil year must necessarily consist of an exact number of days. But the solar year consists of a certain number of days and of a part of a day, (art 214) Hence an artifice is necessary to keep the commencement of the different seasons, as nearly as possible, in the same place of the civil year: that is, if the sun enter the equator on the 20th of March in one year, that it may always enter it on the same day, or nearly on the same day, and that the solstices may be always as nearly as possible on the same day.

The common civil year consists of 365 days. The solar year of 365 days, 5 hours, 48 minutes, and 50 seconds, or 365 days, 6 hours nearly.

It is evident that if each civil year were to consist of only 365 days, the seasons would be later and later every year, and in process of time change through every part of the year

328. In the infancy of astronomy, it was not to be expected that the exact length of the solar year could be obtained with much accuracy, and we find the Egyptians and other nations availing themselves of another method, by which they regulated the times of their agricultural labours. They observed when

Sirius or Arcturus, or some other bright star, after it had been obscured by the splendor of the solar rays, first became visible in the east, before sun-rise This is called the heliacal rising of a star From this time they reckoned a certain number of days to the commencement of the respective seasons of ploughing, of sowing, and of other labours in husbandry.

In this manner they dispensed with an exact knowledge of the length of the year They were ignorant of the precession of the equinoxes, which in a few centuries would have occasioned their rules to fail, or rather to change.

329. The first useful and tolerably exact regulation of the civil year, by help of the solar, took place in the time of Julius Cæsar It was then provided that every fourth civil year should consist of 366 days, and the addition of the day should be made, "die sexto calendas martias," whence the term *bissextile* applied to the year that consists of 366 days · we usually call it leap year, and the additional day is called the 29th of February.

The Calendar so ordered was called the Julian Calendar

330 By the council of Nice, held in the year 325, it was fixed that the feast of Easter, by which the moveable fasts and festivals of the church are regulated, should be the first Sunday after the first full moon, which happened on or after the 21st of March. At that time the equinox happened on the 21st of March. Thus the festival of Easter was intended to be regulated by the spring equinox

At that time it must have been known that the excess of the solar year above 365 days was not quite six hours, and that therefore, in using the Julian Calendar, the equinox would happen sooner every year. There however seems to have been no provision made on that account

The true length of the solar year being less than 365 days, 6 hours, by 11 minutes nearly, the equinox every fourth year was nearly 44 minutes earlier, and in course of time the 21st

of March, instead of being the day of the equinox, might have been the day of the summer solstice Thus the fast of Lent and festival of Easter might have been observed in the middle of summer.

This inconvenience was foreseen before any material alteration had taken place. In the time of Pope Gregory, in 1577, the equinox happened on the 11th of March, or ten days before the 21st It was then determined to remedy the error that had already taken place, and to provide against a future accumulation

It must be generally allowed, that it was right to guard against an increase of the error, but it may be doubted whether a greater inconvenience did not take place to the people in general by correcting the error of the ten days, than if it had remained.

331. The 5th of October, 1582, was called the 15th, and thus the equinox was restored to the 21st of March.

A recurrence of error was prevented in the following manner The true length of the solar year, as far as it was then known from the best tables, founded on the observations of Copernicus, Ptolemy, and Hipparchus, was 365 days, 5 hours, 49 minutes, and 16 seconds. By adding a day every fourth year, in 4 years the addition was $4 \times (10^m\ 44^s)$ too much, or the accumulation of error in 400 years $= 400 \times (10^m\ 44^s) = 2$ days, 23 hours, and 33 minutes nearly. Hence if, instead of making *every* fourth year leap year, every hundredth year for three centuries successively be made a common year, and the fourth hundred year be a leap year, the error in 400 years will be only about 27 minutes, and therefore the error in 20000 years would not be more than a day.

Hence the correction adopted by Pope Gregory, that the years 1700, 1800, 1900, 2100, 2200, 2300, 2500, &c, which, by the Julian Calendar, are leap years, should be common years,

and that the years 2000, 2400, &c. should remain leap years, is quite sufficient The more correct length of the solar year, as now determined, proves the Gregorian correction less exact, but not materially so

332 The Gregorian, or the new style, was not adopted in Protestant countries, till a considerable time had elapsed When it was adopted in England in the year 1752, the error amounted to 11 days This was remedied by calling the 2nd of September, 1752, the 13th

The effect of thus putting, as it were, the seasons backward by 11 days, must at that time have been disagreeable That our mode of reckoning time was made the same as that of other nations, was doubtless a convenience But it might have been more conformable to our climate and the original notions of the festival of Easter, which regulates the other moveable fasts and festivals of the church, if the error that had already accumulated from the Julian Calendar had remained, and the Gregorian correction against future error had been only adopted

The early climate of Italy might have principally induced Pope Gregory to bring back Easter to the regulations of the equinox · and it may have been a powerful motive in Russia for not adopting the Gregorian alteration in the style, that by retaining and suffering the errors of the Julian Calendar to accumulate further, the fast of Lent and festival of Easter will fall at times more convenient in respect to their seasons

The year 1800 having been by the Julian Calendar a leap year, and by the Gregorian a common year, the Russian date is now 12 days behind that of the other countries of Europe.

333. The time of the festival of Easter depends on the first full moon on or after the 21st of March, and therefore, strictly, recourse should be had to astronomical calculation to ascertain the time of Easter for each year But it is sufficient for this

purpose to use the Metonic Cycle, (art 137,) the numbers of which are called Golden numbers.

Short rules and brief tables are given in the Act of Parliament for changing the style, and are usually prefixed to the Book of Common Prayer, by which the times of Easter may be found for any number of years to come The computation so made, must sometimes differ from what a more exact calculation would give, and the time of Easter, if exactly computed, may vary considerably from the computations founded on the Metonic Cycle. However, as the latter mode of calculation is prescribed by the Act of Parliament, no inconvenience, from uncertainty as to the time in which the festival of Easter is to be observed, can arise

By exact computation the 1st of April, 1798, should have been Easter Sunday, whereas by the Calendar prescribed it was not celebrated till the Sunday after. Also the 29th of March, 1818, should have been Easter Sunday, instead of the 22nd of March, as found by the prescribed mode of calculation.

CHAPTER XIX.

ON THE DISCOVERIES IN PHYSICAL ASTRONOMY

334 The astronomical knowledge, that existed before the time of Sir Isaac Newton, was derived from long and tedious observations, which had been continued through many ages. The various discoveries, such as the elliptical motions of the planets, the law of the periodic times, the precession of the equinoxes, the direct motion of the apogee of the moon's orbit, the retrograde motion of its nodes, the variation and evection of the moon, were apparently so many unconnected circumstances.

It was Newton who first, from a few general laws of matter and motion, by help of mathematical principles, shewed the origin and connexion of these different phænomena, and that they were simple results of the general properties which the Creator has ordained should belong to matter and motion Before his time Physical Astronomy did not exist The attempts of Kepler, Des Cartes, and others, to explain several astronomical phænomena from physical principles, now scarcely deserve notice.

335 It would be incompatible with the plan of this work to enter into any detail of the mathematical principles of physical astronomy But the discoveries in physical, are so connected with plane astronomy, and so important, that it was not possible to avoid the mention of many of them, when occasion offered, and it may not be deemed improper to conclude with

a short account of the general advantages, the science of astronomy has received from the application of physical principles

Sir Isaac Newton has shewn that all the bodies of the solar system mutually attract each other That the gravitation or the force of attraction exerted by, or toward any body, is in proportion to the mass of the attracting body That this force, is greater or less, according as the distance from the attracting body is less or greater, and that in proportion to the square of the distance

336 Of the immediate cause of gravitation, he confesses himself ignorant He says,[a] that gravity must be caused by an agent acting constantly according to certain laws but whether this agent be material or immaterial, he did not attempt to decide. He reflected much on this subject, but it does not appear that he ever came to any conclusion which satisfied him self. At this day we are not advanced one step farther toward the knowledge of the proximate cause of gravity, than Newton himself had advanced.

The knowledge of the proximate cause, however, is not necessary to ascertain the existence and laws of the action of gravity The latter are collected from a variety of facts

From the laws of the action of gravity combined with laws of matter and motion, deduced from observations on terrestrial matter, Newton explained the motions observed in the solar system.

The sun situate in the midst of the planets attracts them all toward itself, while they also attract the sun, but from the greater mass of the sun, the effect of the planets in moving the sun is very small, compared with the attraction of the sun on the planets.

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[a] Letter to Dr Bentley, page 438, vol 1 Horsley's Edition of Newton's Works

Had no other impulse been given to each of the planets, they and the sun would have come together in consequence of their mutual attraction. But a proper impulse was given to each planet in a direction either perpendicular, or nearly perpendicular to a line joining the sun and planet In consequence of this impulse, and of the attraction of the sun, each planet continues to revolve round the sun in an elliptical orbit not differing much from a circle, that is, not very eccentric These impulses must have been given at the creation These impulses required, to use the words of Newton,[a] "the Divine Arm to impress them according to the tangents to their orbits"

The simple laws of matter and motion, which the Almighty has been pleased to ordain, are sufficient to preserve the motions of the system for a length of time, to which our bounded intelligence cannot put a limit

337. The preparatory steps of Newton consist, principally, in shewing, that a body projected, and attracted to a fixed centre, describes equal areas in equal times, about that centre, and in investigating the laws of the variation of the force by which a body attracted toward a given point, may be made to move in a given curve

He particularly shews by an interesting application of mathematical principles, that a body moving in an ellipse and describing equal areas in equal times, about one of the foci, must be attracted toward that focus, by a force varying inversely as the square of its distance from the focus· that the squares of the periodic times of bodies, moving in different ellipses about a common centre of force in the common focus, are as the cubes of the greater axes.

He also, conversely, proves that a body attracted to a fixed

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[a] Third Letter to Dr Bentley

centre, by a force varying inversely as the square of the distance, and projected in a direction, not passing through the centre, with a velocity, not exceeding a certain limit, will describe an ellipse about the fixed centre. The increase, or decrease of velocity, generated by the attractive force, is so exactly combined with the velocity of projection, that the efficacy of the attractive force in drawing it from the tangent of the curve, in which tangent it would continue, were the attractive force to cease, is such as always to retain it in the circumference of the ellipse.

After considering a variety of cases about a fixed centre, he considers two or more bodies, mutually attracting each other.

He also demonstrates that if a globe consist of particles each of which attracts with a force varying inversely as the square of the distance, that the united forces of all the particles, compose a force tending to the centre of the globe, and varying inversely as the square of the distance from the centre of the globe.

338 The application of his investigations to the system of the world, may be briefly stated as follows.

The effort by which all bodies within our reach, tend toward the surface of the earth, we call *gravity*. If left to themselves, bodies fall toward it in a right line, but if projected, they tend toward it in a curvilinear course

By gravity also a pendulum, when removed from a vertical position, tends to it again, and so vibrates.

Experiments on the motions of falling bodies and the vibrations of pendulums, after proper allowances made for the resistance of the air, shew that this force of gravity, measured by the velocity produced in a given time, is nearly the same in the same place, at any distance from the surface to which our experiments can reach

But along with the knowledge of this fact, we also arrive at another, of great importance, viz. that however dissimilar bodies are in their visible properties, yet they are all equally affected

by gravity, that each particle of a body is acted upon by the same force, that the component parts of air and gold, are equally impelled toward the earth. This knowledge is derived from observing that all bodies, at the same place, describe, in falling toward the earth, equal spaces in equal times, abstracting from the resistance of the air

To these laws of gravity, we are enabled also by experiment to add a third; that the gravitation toward the earth is the united effect of gravitation toward its separate parts, or that each particle of matter attracts, from whence it follows, that the attraction of gravitation between terrestrial matter is mutual [a] Several strong arguments induced Newton to adopt this, as an Hypothesis, but it seems not to have been fully verified, till long after his death. No facts, proving it, were known to him

Although we are ignorant of the cause of gravitation, yet we can inquire whether it be a principle which has no other connexion with the earth, than that of impelling bodies toward its centre, or whether it be a principle attached to each particle of matter, and so whether the force by which bodies are impelled towards the centre of the earth, arises from the joint attractions of the particles of which the earth is composed Newton, as

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[a] If the Earth had been originally a fluid of uniform density, it would have followed from the mutual attraction of its parts, and from its rotation on its axis, that the increase of the length of a pendulum vibrating seconds would have been nearly as the square of the sine of latitude Also if the Earth had been originally a fluid of unequal density, the denser parts would have so arranged themselves towards the centre, that the law of increase of the length of the pendulum would still be as the square of the sine of latitude Now we know that the interior of the Earth is denser than the surface, and a great number of experiments have shown, that in both hemispheres the increase of the length of the pendulum is as the square of the sine of latitude From hence it has been inferred that the Earth was originally in a fluid state.

The above is one of the results of analysis and experiment that, according to Laplace, ought to be ranked among the few certainties that Geology furnishes.

was said, adopted the hypothesis of this latter mode of the action of gravity; but it was not necessary for his theory, that gravity should arise from the gravitation toward the particles of each body

339 The effect of the attractions of the mountains in Peru, on the plumb line, observed when the measurement of an arc of the meridian was carrying on, was the first direct proof Some circumstances, however, made the result of the experiments dubious. But it was fully verified by the experiments of Dr Maskelyne on the attraction of Mount Schehallien in Scotland (Phil. Trans 1775) It has since been confirmed by Mr Cavendish's experiments on the effects of the attraction of balls of lead, (Phil Trans 1798)[a]

The experiment of Dr Maskelyne was made by observing the effect of the attraction of the mountain in drawing the plumb line of a zenith sector from a vertical position toward itself The observations being made with the utmost care and accuracy, as might be expected from the long experience of Dr. Maskelyne, the result was, that the difference of latitude from measurement was less by 11″, than by observation of the difference of the zenith distances of the same star. Thus the attraction of the mountain, occasioned the plumb line to deviate about $5\frac{1}{2}''$ from a vertical situation.

340 Before the time of Newton, it appears that several

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[a] Laplace has shewn that the effect of the attraction of the excess of matter at the equator, causes two equations in the moon's motion, one in latitude and the other in longitude. The quantities of these equations, having both been well ascertained by an examination of a very great number of observations, have served to deduce the excess of the equatoreal above the polar diameter Each equation gives the same excess very nearly, viz $\frac{1}{305}$. Laplace also has shewn that the excess of matter at the equator of Jupiter occasions certain equations in the motions of the Satellites

eminent men had notions respecting a mutual attraction in the system Kepler in his work "De Stella Marte," speaks of the mutual gravitation of the earth and moon. He says that if they were not retained at their proper distances, the earth and moon would come together, the moon coming over 53 parts of the distance, and the earth over one part He also seems aware, that not only the tides are caused by the attraction of the moon, but also that the irregularities of the moon are caused by the united actions of the sun and earth. But it does not appear that either he, or any other person before Newton, had an idea that the force of gravity toward the earth, combined with the projectile velocity, retained the moon in her orbit, or any notion of the variation of gravity, at different distances from the earth.[a]

Kepler, although an excellent mathematician, seems not to have been able to apply that science to his ideas of gravitation, and Galileo had the merit of first applying the principles of mathematics to investigate the effects of gravity at the earth's surface. He first shewed that a projectile acted upon by the uniform force of gravity, in parallel lines, describes a parabola.

We find no mathematician between him and Newton pushing the inquiry farther and investigating the curve in case of a projectile taking such a range, that gravity could no longer be considered to act in parallel lines About the time however that Newton applied himself to these inquiries, we see several mathematicians considering the laws of action by which bodies may revolve with uniform velocities in different circles about the same centre

We are told that an accidental circumstance first led Newton

[a] It may be considered as a curious circumstance that Galileo computes how long a body would take to fall from the moon to the earth He supposes the force of gravity to continue the same throughout the whole distance

to consider the effects of the gravity of the earth, at a distance from the surface, and to inquire whether that gravity did not extend to, and retain the moon in her orbit the moon by the continual action of this force being drawn from a rectilineal course, and made to revolve about the earth in a nearly circular orbit

To examine this point, he was enabled, from knowing the moon's distance from the earth and its periodic time, to compute how much it deviated, or was drawn from its rectilineal course in one minute, which he found to be nearly 16 feet. He thus found that a force tending to the earth existed at the distance of sixty semidiameters, which impelled the moon toward the earth, 16 feet in one minute The next inquiry was whether this force were constant or variable at different distances from the earth, or rather what was the law of its variation. He saw that if it increased as the square of the distance decreased, at the earth's surface, it would impel a body 3600 × 16 feet in one minute, or 16 feet in one second. This is the space a body falls by gravity in one second Hence he concluded that the force of gravity, diminished in the duplicate proportion of the semidiameter of the earth, to the moon's distance, was the force acting at the moon and retaining it in its orbit.

341. We deduce somewhat more easily the law of gravitation towards each of the planets, which have satellites. It is found that the satellites of Jupiter move round Jupiter in orbits nearly circular, and that the squares of the periodic times are as the cubes of their distances from the primary. Whence it may be easily shewn, that they are constantly impelled toward or attracted by Jupiter, by a force increasing as the square of the distance from Jupiter decreases. The same may be said of Saturn and the Georgium Sidus Here then are the Earth, Jupiter, Saturn, and the Georgium Sidus, each attended with an attractive influence, acting by the same laws, and therefore, by

analogy, we may justly conclude, that the remaining planets attract by the same laws.

342 Newton's investigations of the motions of bodies about the same centre of force, combined with Kepler's discoveries, prove that each of the planets is attracted toward the sun, by a force varying inversely as the square of the distance from the sun.

For Kepler shewed that each planet moved in an ellipse, and described equal areas in equal times about the sun in the focus, and that their periodic times were as the cubes of the greater axes of their orbits. Newton demonstrates, that when this takes place the law of attraction is as above stated.

343. Thus then by the moon we ascertain that the earth exerts an attractive influence; by the satellites of Jupiter, Saturn, and the Georgium Sidus, that these planets exert a similar influence, and by the forms of the planetary orbits and laws of motion in those orbits, that the sun also possesses an attractive force We find the law of action is the same in all the attracting bodies. But if we examine farther we find the forces exerted very different at the same distance from each body. If we compute the force exerted by the earth, at the distance of the sun, by diminishing the force of Gravity at the earth's surface in the duplicate proportion of the semidiameter of the earth to the sun's distance, we shall find it small indeed, compared with the force the sun exerts on the earth.[a]

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[a] The masses and densities of those planets, which have satellites, have been ascertained by their attractive actions on the Satellites The density of water being called unity, then nearly,

the density of the Sun	=	1,3
of the Earth	=	5,0
of Jupiter	=	1,0
of Saturn	=	0,6
of Georg Sidus	=	1,5

344 As the planets are attracted toward the sun and attract their satellites, we may conclude that they attract one another and the sun Also, as we find the attraction of the earth made up of the attractions of its parts, so we may conclude the attractions of the sun and planets composed of the attractions of their parts, and that the law in the system is, that every body attracts with a force at a given distance in proportion to its mass, and that the force diminishes as the square of the distance of the attracted body is increased

345 As soon as Newton had published his discoveries, there could be no rational doubt of this being the law which exists throughout the solar system, and every step, that has since been made in Physical Astronomy, has furnished additional proofs

It is probable that Newton derived no assistance in the discovery of the law of gravity · yet he does not seem unwilling that others should have a share in the merit. He ingenuously tells us that Wren, Hook, and Halley had separately discovered from Kepler's law of the periodic times, the law of attraction towards the sun, if the planets moved in circular orbits. But the great fame of Newton rests not upon this foundation, that he merely discovered the law of gravity. He proceeded by synthesis to examine the phænomena that would offer themselves in a system so regulated His transcendent mathematical powers enabled him to point out the origin of all the more splendid discoveries of former ages He shewed that the planetary orbits must be elliptical, that the lunar irregularities, the precession of the equinoxes, and the phænomena of the tides must take place from the principle and law of universal attraction.

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the density of Venus is supposed to be somewhat greater than the density of the earth from the effects of that planet in diminishing the obliquity of the ecliptic, and by changing the plane of the earth's orbit

thereby evincing, in the strongest manner, that he had arrived at the knowledge of those laws, which the Creator had willed for upholding the system of the world

346. Newton had extended the boundaries of mathematical knowledge, as much as he had those of physical. He preferred exhibiting his investigations and conclusions in a geometric, rather than in an analytic form, as better suited to the general outlines of physical astronomy, and also as better adapted to call the attention of the world to his great discoveries To extend the limits of physical astronomy, and to explain discoveries that have been made by comparing modern and ancient observations, it has been found necessary to adopt entirely the analytic method.

A considerable time elapsed from the publication of Newton's Principia in 1687, before any attempt was made to extend the investigations of Newton In 1740 Maclaurin, Euler, and Bernoulli shared a prize given by the Royal Academy of Sciences at Paris, for their dissertations on the tides, in which they made considerable advances in the path pointed out by Newton

Soon after, Euler, D'Alembert, and Clairaut, engaged in the famous problem of the three bodies, as it has been called That is, to investigate the motions of three bodes, acting upon each other according to the laws of gravity The problem in its general extent is far beyond the powers of analytics in their present state : but in the case of the sun, earth, and moon, we can approximate to the solution with sufficient exactness. For, the sun disturbs the motions of the moon, as seen from the earth only by the difference of its attractions on the moon and earth, which difference is always very small, compared with the force by which the moon is attracted towards the earth

The importance of an exact knowledge of the lunar motions in finding the longitude at sea, seems principally to have incited

the exertions of these mathematicians A difficulty soon occurred which made them at first doubt of the exactness of the Newtonian law of gravity They could not reconcile the mean motion of the lunar apogee, as determined by calculation, with that deduced from observation[a] They saw the latter was double of the former At last Clairaut, by extending his approximations[b] overcame this difficulty, and added a new proof of the law of gravity

347 There were two phenomena, however, to which Flamstead and Halley first called the attention of astronomers, which for many years baffled all attempts to account for them, from the received laws of gravity These were the acceleration of the moon's motion, (art. 230,) and the acceleration of Jupiter's, and retardation of Saturn's motions (art 213.)

Dr. Halley's computations on the ancient observations had been verified by other astronomers, and no doubt remained of the facts The acceleration of the moon's motion had also been verified by computations made on three eclipses observed by Ibn Junis near Cairo, towards the end of the 10th Century.

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[a] Newton himself seems to have long been sensible of this difficulty, and to have exerted himself in the computation without success In the first edition of the Principia he mentions computations by which he had ascertained the agreement nearly of his Theory with Flamstead's Tables, accommodated to the Hypothesis of Horrox But he says, "Computationes autem ut nimis perplexas et approximationibus impeditas, neque satis accuratas, apponere non lubet" In the subsequent editions of the Principia, he does not attempt to reconcile the observed motion of the apogee with his Theory It would be very interesting to know the particulars of his computation

[b] Not considering the eccentricity and inclination of the lunar orbit, the mean motion of the lunar apogee, that of the moon being unity, is expressed by a series of terms of the form $\frac{3}{4}m^2 + \frac{325}{32}m^3 +$ &c. were $m = \frac{\text{periodic time of the moon}}{\text{periodic time of the sun}} = \frac{1}{13}$ nearly Clairaut's first approximation extended only to the term $\frac{3}{4}m^2$. Vide Trans Royal Irish Academy, vol. 13.

348. Euler, who, as a mathematician, ranks so high, directed his attention to the motions of Jupiter and Saturn So early as the year 1748 he published an investigation of them, but failed to explain the difficulty Other mathematicians engaged in the inquiry For a long time the object of their pursuit eluded them, but their exertions tended much towards perfecting physical astronomy

Euler investigated many of the disturbances which take place by the mutual action of the sun and planets. He first shewed that the diminution of the inclination of the ecliptic to the equator, which ancient observations appeared to show, was occasioned by the action of the planets by which the plane of the earth's orbit is gradually changed.

Lagrange, who has become so distinguished by his many splendid improvements in mathematics and mathematical philosophy, about 1765 published[a] his investigations respecting the motions of Jupiter and Saturn

The celebrated Laplace in 1773 shewed that the mean motions and mean distances of the planets were not subject to any variation arising from their mutual actions on each other, or at least were so nearly constant that nothing could appear to the contrary from the most ancient observations Hence the explanation of the acceleration of Jupiter and retardation of Saturn, that Lagrange and others had given, could not be the true one.

Soon after Lagrange himself proved strictly, what Laplace had proved only by approximation, that neither the mean motions, nor mean distances of the planets were subject to any perceptible alteration from their mutual attraction.

It was not till 1786 that Laplace discovered the true explanation of the difficulties as to Jupiter and Saturn, after it had

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[a] Turin Memoirs Vol 3

been sought for in vain, above thirty years, by the continued exertions of the first mathematicians

His investigations furnished another confirmation of the mutual attraction of the system. He shewed that the quantity of acceleration in the motion of Jupiter and retardation in that of Saturn deduced from computation, agreed with observation [a]

349 The difficulty of the acceleration of the moon's motion yet remained, and it had fully as much occupied the attention of those who endeavoured to improve Physical astronomy, as that just mentioned

The Royal Academy of Sciences at Paris had proposed it several times as the subject of their prize It had eluded the

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[a] It is not easy to make his discovery intelligible to those not conversant in the computations of physical astronomy The *equations* arising from the mutual attraction of the bodies of the system, are divided into *secular* and *periodical* In fact it is now known, that all equations are periodical, but the term, 'secular' distinguishes those that do not depend on the position of the *bodies* as to each other As these equations appertain to a long period, they are called secular. Periodical equations are those that depend on the position of the bodies to each other

Thus the variation of the moon (art 231) depends on the angular distance of the moon from the sun, being proportional to the sine of twice the angular distance of the moon from the sun, and is called a periodical equation The acceleration of the moon's motion not depending on the positions of the sun, moon, and earth, is called a secular equation

Lagrange had at first conceived, that the acceleration of Jupiter was from a secular equation, but Laplace, and then he himself, shewed that no such equation could exist in the planetary motions Therefore Laplace was led to look for a periodical equation, and he observed that as twice the mean motion of Jupiter was very nearly equal to five times that of Saturn, an equation of a very long period would result from thence, which might be sensible To investigate this, it was necessary to extend the approximations to a greater length than had hitherto been done. It might, and, it is likely it did occur to others before this time, that this was a probable source of the phenomena, but till the existence of a secular equation had been disproved, the formidable calculations might have deterred. Equations depending on the difference between five times the motion of any planet and twice that of another, actually exist, but are insensible in the other planets.

researches of Lagrange, yet his investigations on the subject gained the prize in 1772 Bossut had endeavoured to explain it by the resistance of an ether, or subtile fluid pervading the whole system Laplace endeavoured to explain it by supposing that the transmission of gravity, like that of light, was not instantaneous, and on this hypothesis he made some important investigations At length in 1787 Laplace himself discovered the true cause; that it was a simple result of the laws of gravity The actions of the planets, besides changing the plane of the earth's orbit, change also its eccentricity The eccentricity now is diminishing, and will continue, for many ages to come, to do so It will afterwards increase, and thus be subject to periodical changes These changes will affect, through the action of the sun, the angular velocity of the moon about the earth, and hence at the present time an acceleration takes place. Nothing can be more satisfactory than the results of Laplace on this subject. He has shewen that the mean motions of the apogee and of the node are affected by the same cause, and it appears that the quantities assigned by computation agree with the results arising from a comparison of ancient and modern observations.

The true cause of the acceleration or secular equation of the moon's mean motion being discovered, Laplace's former investigations, on the hypothesis that the transmission of gravity was not instantaneous, have served to prove that the transmission of gravity, if not instantaneous, is immeasurably quicker than that of light.

350 The results of all the improvements in physical astronomy, since Newton first called the attention of mankind to it, have been given to the world by Laplace in his great work, entitled "Mecanique Celeste." This and the Principia of Newton will probably be considered by late posterity as the two noblest monuments of human science. The Principia of

Newton was the work of one mind, which could derive no assistance from those who had gone before The energies of the most distinguished abilities had been for many years employed in collecting materials for the fabric that Laplace has erected Newton and Lagrange have assisted in an eminent degree. Maclaurin, Euler, T Simpson, Clairaut, D'Alembert, and others, greatly contributed Laplace himself, besides the merit of planning, and of selecting, and arranging the materials, has the honor of having executed many of the most difficult and highly finished parts of this great work.

351. No motion is now known to exist in the system, but what we can shew to be conformable to the laws of universal gravitation.[a] The mean motions and mean distances of all the planets are to be considered invariable, and the effects of their mutual actions are all periodical We can now ascertain for thousands of years the state of the system, should such a continuance be permitted by the Divine Author.

The obliquity of the ecliptic, which now is diminishing by a small quantity every year, will never be diminished by more than a degree or two. This is a very interesting result Had the obliquity continued to decrease, the equator at last would have coincided with the ecliptic, and a great part of the earth would have been rendered incapable of producing the necessary food for the existence of men and other animals.

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[a] It must not however be supposed that the analytical science, as applied to physical astronomy, is perfect, or even in a state approaching to perfection Notwithstanding the great progress that has been made during the last century, much remains to be done Because the orbits of the planets are inclined at small angles to the ecliptic and to each other, and because the eccentricities of the orbits are small, we are enabled with tolerable facility to compute by approximation the disturbing effects of the planets on each other But it will be a work of great labor and difficulty to compute the disturbances of the new planet Pallas, because its orbit is so much inclined to the orbits of the other planets

352 In all our inquiries into the operations of nature, by which should always be understood, the modes of existence and laws assigned to the objects of the creation by the Divine Creator, we meet with sources of delight and admiration, but in none more, than when in contemplating the objects of astronomy

The magnitudes and the distances of the bodies of the solar system when measured by our ideas so vast, the immense number of the fixed stars placed at immeasurable distances from us, and from each other, shew us the magnificence of the creation.

By the discoveries of Newton we are permitted, as it were, to understand some of the Counsels of the Almighty. From these we can, by demonstration, overturn the absurd doctrine of blind chance. We see that a Supreme Intelligence placed and put in motion the planets about the sun in the centre, and ordained the laws of gravitation, having provided against the smallest imperfection that might arise from time And let us not imagine that only in these vast bodies the Supreme care was employed. Let us not imagine that man, apparently so insignificant, cannot be an object of attention in a world so vast The protecting hand of the Creator is equally visible in the smallest insects and vegetables, as in the stupendous fabrics which astronomy points out to us He, who formed the human mind so different in its powers and mode of existence from the rest of the works of the creation, has assigned laws peculiarly suited to *its* preservation and improvement laws not mechanical, but moral· laws only obscurely seen by the light of reason, but fully illumined by that of revelation

APPENDIX.

The following Problems are intended to illustrate several of the articles in the foregoing treatise, and also to explain the practical mode of solving some of the more useful problems in astronomy

To understand several of these problems, a tolerably extensive knowledge of plane and spherical trigonometry is required, more particularly, a knowledge of the elegant and very useful rules for the circular parts of a right angled spherical triangle, and the analogies for oblique angled spherical triangles, discovered by Napier the inventor of Logarithms A knowledge of the solutions of the cases of oblique angled spherical triangles, not included in Napier's Analogies, and also of many of the trigonometrical expressions for two arcs is required These expressions depend on the fundamental formula

$$\text{Sin}\,(A + B) = \sin A \,.\, \cos B + \cos A \,.\, \sin B$$

where A and B represent any arcs positive or negative

The Astronomical problems given, are of two kinds

1. Problems in the solutions of which great circles of the sphere, only, are used.

2. Problems in which small circles are used, and small variations of the parts of spherical triangles

For the latter, the fluxional or differential calculus, would have, in some few cases, furnished solutions somewhat more concise, but not materially so, in any of the examples given. Oftentimes the differential calculus, applied to trigonometrical formulæ, leads to a

very complicated process, which might be avoided by the consideration of differential triangles

Unnecessary repetition will be avoided by mentioning here, that in the following Problems, radius is unity, and therefore is not put down that when the rules of circular parts are referred to, it is not meant that the equation is put down exactly as given by the rules, but such as may easily be deduced from thence Thus because tang cot $= 1$, the reciprocal of the tangent is sometimes put for the co-tangent, &c

Prop I *To prove that the time from a star's rising to its coming to the meridian is equal to the time from coming to the meridian to setting*

Let PH (fig. 48) represent the meridian, RS the horizon, and RoS the star's path, rising at R and setting at S.

Since RP is equal to PS, and PH common to the two triangles RPH, HPS, by the rules for circular parts we shall have obviously the angles RPH, HPS, equal to each other, and therefore the angles RPo, oPS are equal, and hence the time of describing Ro equal to the time of describing oS, since the motions are equable

For the sun or moon the angles RPo, oPS are not equal, because the polar distances RP, PS are not equal For the sun the forenoon exceeds the afternoon from midsummer to midwinter; and from midwinter to midsummer the afternoon exceeds the forenoon.

Prop II *Given the sun's longitude, to find its right ascension and declination*

In fig. 49 ES represents the ecliptic, ED the equator, and SD a circle of declination

By the rules for circular parts

tan ED = cos E tan ES

and sin SD = sin ES sin E

or tan R Ascens = cos Ob. Ecl tan Long

sin Decl. = sin long sin Ob Ecl,

When the tang of longitude is negative, the tang of R Ascens will be so likewise, and the R Ascens and long will be always in the same quadrant, as is otherwise evident

The sine of declination has the same sign as the sine of long, and therefore between 180° and 360° of longitude will be south, as is otherwise obvious These remarks are only made here to call the attention to the signs of the quantities.

The sun's longitude at any time is found by the solar tables, and is also given in the Nautical Almanac for every day at apparent noon at Greenwich, hence may be found for any given time, at any place, the longitude of which is known Therefore the obliquity of the ecliptic being known, the right ascension and declination will be had as above.

The mean obliquity of the ecliptic for 1800 was 23° 27′ 57″ and it diminishes 0″, 45 every year For the mode of finding the true obliquity from the mean, see Prop 18

Prop III. 1 *To find the time of sunrise in a given latitude on a given day* 2 *When due East* 3 *The time of its being at a given altitude.*

In fig 50 the circles of the sphere are supposed to be seen from a point in the continuation of a radius, at right angles to the plane of the meridian, and therefore the horizon, equator, and prime vertical appear right lines, as HO, EQ and ZN Dd is the sun's parallel of declination R the place of the sun when rising, V the place when on the prime vertical or due east, and FB the given altitude

On a given day, the sun's declination may be found as in the preceding problem, or may be deduced from the Nautical Almanac, where it is given for every day at noon at Greenwich

1. RPO = hour angle from midnight, R being the place of the sun at rising

From the right angled triangle RPO, by circular parts, cos RPO = cot RP . tan PO.

or cos hour angle from midnight = tan decl tan lat

When the declination is south, its tangent is negative, and therefore the cos hour angle is negative, and the angle is greater than 90° The hour angle being reduced to hours, &c by dividing by 15 gives the time of rising

2 To find ZPV From the right angled triangle ZVP, by circular parts

Cos ZPV = cot PV tan ZP, or

Cos hour angle from noon = tan decl cot lat

3 In the oblique angled triangle, BZP, are known the three sides to find the angle BPZ Let P = BP + ZP + BZ By spher. trig[a] sin BP sin ZP . sin $\frac{1}{2}$P . sin ($\frac{1}{2}$P — BZ) ∷ rad² : cos ² $\frac{1}{2}$ ZPB Hence ZPB is found, and therefore the time from noon.

When $\frac{1}{2}$ ZPB is small it cannot conveniently be found with accuracy, from its cosine, because the cosines of small arcs vary very slowly. The following analogy will then serve[a] sin BP . sin ZP sin ($\frac{1}{2}$P — BP) sin ($\frac{1}{2}$P — ZP) . rad ² . sin ² $\frac{1}{2}$ ZPB

The first analogy is to be used when ZPB is greater than 90°, and the second when less

If great accuracy be desired, the effect of refraction should be considered by correcting the altitude Also the sun's declination should be exactly computed

The effect of refraction on the rising of the sun will be afterwards investigated

This problem contains the computations mentioned in articles 53 and 292

Prop IV *Given the right ascension and declination of an object, to find its latitude and longitude.* Art (15).

In fig 51, let P and N be the poles of the ecliptic and equator, and S the object

If the Right Ascension be > 0° < 90°, PNS = 90° + R. A
.... > 270 < 360 , PNS = R. A — 270
.... > 90 < 270 , PNS = 270 — R A

[a] See Luby's Trig 2d edit. art. 135 p 81.

$$NS = 90^\circ \pm \text{decl.}, \quad PN = \text{obliq of ecl}$$

By spherical trig

$$\cos PS = \cos PN \cos NS + \sin PN \sin NS \cos PNS$$

Hence may be deduced*

$$\sin^2 \tfrac{1}{2} PS = \sin (\tfrac{1}{2}(PN + NS) + M) \sin (\tfrac{1}{2}(PN + NS) - M),$$

When M is an auxiliary arc, such that

$$\sin^2 M = \sin PN \sin NS \cos^2 \tfrac{1}{2} PNS$$

The application of the tables of logarithms to find M and then PS, the distance from the pole of the ecliptic, is very simple, and no distinction of cases occurs

The angle SPN can now be found from the sides of the triangle SPN, as in Prop III

$$\text{R A} > 270^\circ < 90^\circ \begin{cases} \text{If SPN} < 90^\circ \ldots\ldots \text{long} = 90^\circ - \text{SPN} \\ \ldots\ldots > 90 \ \ldots\ldots \text{long} = 450 - \text{SPN} \end{cases}$$

$$\text{R A.} > 90 < 270 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{long} = 90 + \text{SPN}.$$

The above solution is rather longer than those usually given, but it has the great advantage of not being embarrassed by a difficult distinction of cases PS having been found, SPN might have been found from the simple theorem of the sines of the sides being as the sines of the opposite angles; but then an ambiguity would have arisen Now this latter theorem may be made use of for verifying the computation Thus

$$\log \sin SN + \log \sin SNP = \log \sin SP + \log \sin SPN,$$

or, as readily follows,

$$\log \cos \text{decl} + \log. \cos \text{R A} = \log \cos \text{lat} + \log \cos \text{long}$$

Prop V *To find at any time the height and longitude of the nonagesimal point*

The nonagesimal point or degree is that point of the ecliptic 90° from the horizon, and is therefore the highest point of the

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* See Trig art. 137, p 81, 2d method

ecliptic The continuation of the notice of this point by astronomers has been on account of its use, in computing the parallax of the moon in latitude and longitude in eclipses

Let HO, (fig 52) represent the horizon, HNO the ecliptic, P the pole of the ecliptic, and Z the zenith Draw through P, Z the great circle PZN, and N is the nonagesimal point, for the right angled triangles HZN and NZO are equal, and therefore HN = NO. Now ZN the co-altitude of N, is the latitude of the zenith point, and the longitude of Z is the same as the longitude of N

Hence if we find the latitude and longitude of the zenith point, we have the co-altitude and longitude of the nonagesimal Now at a given time we know the right ascen of the zenith point, which is the right ascen of the meridian or the sidereal[1] time, and the declination of the zenith is the latitude of the place Hence this problem is contained in the last

The introduction of the term nonagesimal was unnecessary, because the latitude and longitude of the zenith points will serve more conveniently the purposes of the height and longitude of the nonagesimal.

Prop VI *To find the time of rising of a given star on any day in a given place*

The sun's right ascens (S) is given in the Nautical Almanac, or may be found as in Prop II Let F represent the right ascension of the star

Then S—F will be the angle at the pole of the equator between the star and sun

Compute as in Prop 3, (fig 50), by help of the star's declination and latitude of the place, DPR the angular distance of the star from the meridian at rising Then the angular distance of the

[1] The sidereal time is found from the mean time, reckoned from the preceding noon, by increasing the mean time in the ratio of 366 365, and adding the sun's mean longitude at preceding noon.

sun from the meridian when the star is rising $= DPR + S - F$, which reduced into time at the rate of 15° for one hour will give the time of rising When $DPR + S - F$ is negative, or greater than 180°, the sun, at the rising of the star, is to the west of the meridian, otherwise to the east.

For great accuracy an allowance should be made for refraction, and also the sun's right ascension should be computed for the time of rising Therefore when the time is found with the sun's right ascension at noon, the sun's right ascension should be computed for the time of rising, found nearly, and then the operation repeated

Remark $\frac{DPR}{15}$ reduced to the solar time by diminishing it in proportion in the ratio of 365 . 366 will give the interval between the rising of the star and its passage over the meridian Hence if its passage over the meridian be known, its time of rising will be known. The times of the passages of the planets over the meridian of Greenwich, and the declinations, are given in the Nautical Almanac, therefore the times of rising or setting, may in this manner be found nearly

Prop VII. *To find the time of the rising of the moon on a given day*

The right ascensions and declinations of the moon are given in the Nautical Almanac for noon and midnight at Greenwich, and therefore may be found by proportion for any given place If we knew the right ascension and declination of the moon at rising, the problem would be solved exactly the same as the last problem. But we cannot exactly know the right ascension and declination of the moon at its rising without knowing the time of rising. This is the inconvenient circumstance in this problem But the difficulty is obviated by using the right ascension and declination at the noon or the midnight at Greenwich nearest the time of rising, and then finding the time of rising, as for a star (Prop 6.) With this time find, by the help of the difference of longitude, the corresponding.

time at Greenwich, and thence the right ascension and declination of the moon. With these find the time of rising more accurately. In like manner a third operation might be used, but this will scarcely be necessary.

The horizontal parallax of the moon being always greater than the horizontal refraction, the rising of the moon is retarded. The computation of the quantity will be shewn in a subsequent problem.

Prop. VIII. *To find when a star rises heliacally* (art. 327.)

A star of the first magnitude rises heliacally, or first becomes visible after having been obscured by the solar rays, when the sun is about 12° below the horizon.

Let F (fig. 53) be the star rising; DA the right ascension of the star, A being the first point of aries; AXL, the ecliptic; and the arc KL perp. to the horizon $= 12^\circ$.

Then L is the place of the sun, when the star rises heliacally. From the right angled triangle CED by circular parts

Sin CD = tan lat. tan decl. Hence, from the right ascension of the star, AD, AC is known in the triangle ACX, and also the adjacent angles A and C are known.

By Napier's Analogies[a]

$$\cos\tfrac{1}{2}(C+A) : \cos\tfrac{1}{2}(C-A) :: \tan\tfrac{1}{2}(AC) : \tan\tfrac{1}{2}(AX+CX)$$
$$\sin\tfrac{1}{2}(C+A) : \sin\tfrac{1}{2}(C-A) :: \tan\tfrac{1}{2}(AC) : \tan\tfrac{1}{2}(AX-CX)$$

Hence AX is known, and therefore the angle X is easily found.

$$\sin XL = \frac{\sin KL\,(12^\circ)}{\sin X}.$$

Whence AL the sun's longitude is known, and therefore the day when the heliacal rising takes place.

To investigate the heliacal rising of a star 2000 years ago, we must decrease the present longitude of the star by $2000 \times 50''$, $= 28^\circ$ nearly. Then with the longitude and latitude of the star, and

[a] Luby's Trig. art. 137, p. 83.

obliquity of the ecliptic, compute the right ascension and declination,[a] and then as above find the sun's longitude at the heliacal rising, it will be sufficiently exact to find by common proportion the number of days from the equinox from the longitude of the sun thus found, taking 59′.8″ for the motion in 24 hours[b]

Prop IX. *To find the sun's declination when the twilight is shortest in a given latitude.*

Let R*r* (fig 54 1) represent a parallel circle 18° below the horizon HO (art 52). KNTYL a great circle touching R*r* in T and intersecting the equator EQ so that the angle Y = HCE (the co-latitude). Draw any parallel of the equator MNS When the sun is in this parallel the arch MN is described in the same time, as the arch CY when the sun is in the equator. For let the arches MW and NX be perpendicular to the equator, then the right angled triangles MWC and NXY will be equal, and WC = XY, consequently WX = CY Therefore since MN and WX are described in equal times, MN and CY will be described in equal times. Hence the portion AT of a parallel of the equator between the horizon and its parallel R*r* is described in less time than the portion of any other parallel to the equator between the same circles, because the time of describing AT = the time of describing CY = the time of describing MN, less than the time of describing MS Hence the twilight is shortest when the sun describes the parallel AT, that is, when the sun's declination is TI

Now, because KTL touches R*r*, the vertical circle ZFT is at right angles to KTL. Hence in the right angled triangles CFB and BTY, the vertical angles are equal and C = Y, therefore these triangles are equal, TB = BF and each = 9°. Also DF at right angles to EQ = TI the declination Hence if we conceive the cir-

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[a] Converse of prop 4

[b] The latitude of the star and obliquity of the ecliptic should also be reduced to what they were 2000 years ago, but this degree of accuracy would be quite unnecessary, in regard to any use that could be made of the result

cles of the sphere projected perpendicularly on the plane of the meridian, and FG be drawn perpendicular to HO, and to meet EQ in G, FG will be the tangent of FB, and FD = the sine of the declination, and DGF = the latitude. Therefore by plane trigonometry

$$\text{rad} : \sin CGF :: FG : FD, \text{ or}$$
$$\text{rad} : \sin \text{lat} :: \tan 9^{\circ} : \sin \text{declination},$$

when the twilight is shortest.

This Prop. may be resolved in somewhat a more general form, as follows:—

To find the declination of the sun or star, when the time of change from a given altitude A *to a given altitude* B *is the shortest possible.*

In fig. 54. 2, L and M are the places of the object when the angle LPM is the least possible, the given zenith distances being LZ and ZM. Considering the triangles ZLP and ZMP, and the differential triangles that may be formed at L and M, it readily appears that the angles ZLP and ZMP are equal, and thence that MZP and LZP are the supplements of each other, as their sines are equal. Their sines are equal because the sides LP and MP are equal, and the side ZP common to the two triangles. The same may also be readily deduced as follows, but not so simply as by the differential triangles. Indeed this is an example, among many others, of the extreme facility of obtaining results by differential triangles as compared with the manner of obtaining the same by trigonometrical formulæ. By trigonometry

$$\cos ZM = \cos ZPM \cdot \sin ZP \cdot \sin PM + \cos ZP \cdot \cos PM.$$

And since ZM and ZP are constant, the differential equation is

$$0 = -\sin ZPM \,.\, \sin ZP \,.\, \sin PM \,.\, d.ZPM + \cos ZPM \,.\, \sin ZP \,.$$
$$\cos PM \,.\, d.PM - \cos ZP \,.\, \sin PM \,.\, d.PM.$$

therefore

$$\frac{d.ZPM}{d.PM} = \frac{\cos ZPM \cdot \sin ZP \cdot \cos PM - \cos ZP \cdot \sin PM}{\sin ZPM \cdot \sin ZP \cdot \sin PM}$$

$$= -\frac{\cos ZMP \cdot \sin ZM}{\sin ZPM \cdot \sin ZP \cdot \sin PM}$$

$$= -\frac{\cos ZMP \cdot \sin ZP \cdot \sin ZM}{\sin ZMP \cdot \sin ZP \cdot \sin ZM \cdot \sin PM}$$

$$= -\frac{1}{\tan ZMP \cdot \sin PM}.$$ [a]

Also for the same reason

$$\frac{d \cdot ZPL}{d \cdot PL} = -\frac{1}{\tan ZLP \cdot \sin PL}.$$ [a]

Now LPM is a *minimum*, and therefore

$$d \cdot LPM = d \cdot ZPM - d \cdot ZPL = 0,$$

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[a] This immediately appears from the differential triangles, but even the consideration of this expression is unnecessary, as the equality of the differential triangles at L and M is at once seen.

Note by the Editor.—The prop. in its general state may be solved very simply as follows. Let the object cross on the parallel bm, fig. 51. 3. then the angle mPb is to be a *min.* Make ZPs on the sphere's surface equal to mPb, and $Ps = Pb$, and draw the arc of a great circle Zs, and join sm. Then since ZP is constant, and the angle ZPs a *min.* the arc Zs must be a *min.*; but the spherical triangles bPZ, mPs being obviously in every respect equal, we have $sm = Zb$, and therefore given. Hence in the spherical triangle smZ, the two sides sm, mZ are given, and the third side to be a *min.* This will obviously be the case when sm, mZ coincide. The object must consequently describe such a parallel nv, that Zn, sn may coincide.

We may from this readily compute as follows:—let fall the perpendicular Px, then xn is obviously half the sum, and xZ half the difference of the given zenith distances, which we shall call z and z'. From the right angled triangles xPZ, xPn, we have

$$\frac{\cos PZ}{\cos \frac{1}{2}(z-z')} = \cos Px = \frac{\cos Pn}{\cos \frac{1}{2}(z+z')},$$

whence

$$\sin \text{decl.} = \sin \text{lat.} \times \frac{\cos \frac{1}{2}(z+z')}{\cos \frac{1}{2}(z-z')}.$$

For shortest twilight $z = 108^\circ$ and $z' = 90^\circ$, hence $\cos \frac{1}{2}(z+z') = -\sin 9^\circ$, and $\cos \frac{1}{2}(z-z') = \cos 9^\circ$, therefore, &c.

This mode of solution exemplifies the advantage of discussing by spherical geometry, previous to converting a question into formulæ.

or $d.\,ZPM = d.\,ZPL$, also $PM = PL$, and $d\,PM = d\,PL$, consequently, $\tan ZMP = \tan ZLP$, and these angles are equal.
Hence

$$\sin MZP = \frac{\sin ZMP \,.\, \sin MP}{\sin ZP} = \frac{\sin ZLP \,.\, \sin LP}{\sin ZP} = \sin LZP$$

Therefore one of these angles must be the supplement to the other, and consequently $LZC = CZM$.

Conceive the circles of the sphere orthographically projected on the plane of the meridian. Then $LK = \sin LZ \,.\, \sin LZK$, and $IM = \sin MZ \,.\, \sin IZM$.
Also $oK = \frac{IK \,.\, LK}{LK + IM} =$ (since $LZK = IZM$) $\frac{\cos LZ - \cos ZM}{\sin LZ + \sin ZM}$ $\sin LZ = \tan \frac{1}{2}(ZM - LZ) \,.\, \sin LZ$. And $OC = KC - OK = \cos LZ - \tan \frac{1}{2}(ZM - LZ) \,.\, \sin LZ$, and then OG (the sine of declination) $= OC \,.\, \sin lat$.

For the shortest twilight $LZ = 90°$ and $ZM = 108°$, hence $OC = -\tan 9°$, and therefore $\sin decl\ south = \sin lat \,.\, \tan 9°$.

Prop. X. *To find when Venus is brightest.*

Let S, T and V, (fig. 55,) be the Sun, the Earth, and Venus when brightest. Let TV also meet the orbit of Venus, supposed circular, in H, and join S, H, and S, V. The brightness of Venus varies as the versed sine of the exterior angle directly, (art. 116,) and as the square of the distance from the earth inversely. For the density of light decreases as the square of the distance from the radiating body increases.

Hence when Venus is brightest $\frac{\text{v}\sin SVH}{TV^2}$ is a maximum, and therefore $\text{v}\sin SHV \times TH^2$ is a maximum, because $TH \times TV$ is constant, and therefore TH varies inversely as TV.

Let $TH = x$, $ST = 1$ and $SH = m$. Then by plane trigonometry,[a]

$$m^2 + x^2 - 2\,m\,x \,.\, \cos SHT = 1.$$

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[a] Luby's Trig. p. 32

and because v. sin SHT $= 1 - \cos$ SHT,

it follows that v. sin SHT $= \dfrac{1 - m^2 - x^2 + 2mx}{2mx}$.

Therefore $\dfrac{1 - m^2 - x^2 + 2mx}{2m} \times x$ is a max.

or $x - m^2 x - x^3 + 2mx^2$ is a max.

Hence by the principles of the diff. calc.

$$1 - m^2 - 3x^2 + 4mx = 0,$$

$$\text{or } x^2 - \frac{4mx}{3} = \frac{1 - m^2}{3}.$$

This equation gives

$x = \dfrac{2m}{3} \pm \frac{1}{3}\sqrt{3 + m^2}$. The upper sign can only be used, because $\sqrt{3 + m^2}$ is always greater than $2m$.

By this value of x the angle STV will be found about 40° for Venus (art. 110), and the point V of greatest brightness lies between inferior conjunction and greatest elongation. If the distance of Venus from the sun were $= \dfrac{1}{\sqrt{5}}$, the greatest brightness would be at the greatest elongation, for then $x = \dfrac{2}{\sqrt{5}}$.

Prop. XI. *To find when a planet appears stationary.*

Let S (fig. 56) represent the sun, T and P the earth and planet respectively, their orbits being supposed circular.

When P and T are stationary with respect to each other, the line TP moves parallel to itself, and the angles T and P only vary by the apparent motions of the sun, as seen from the earth and planet: which motions are equal to the angular motions of the earth and planet as seen from the sun.

Now the angular motions of the planets about the sun are to each other inversely as their periodic times, and by Kepler's law, the squares of the periodic times are as the cubes of the distances,

hence the squares of the angular velocities of the planets are inversely as the cubes of their distances.

Let $t=$T, $p=$P, $r=$ST, and $r'=$SP, $t'=$ a small variation in T, and $p'=$ a corresponding small variation in P.

(1) By trig $r \sin t = r' \sin p$,

(2) and $r \sin(t+t') = r' \sin(p+p')$

By equat (2)

$r \sin t \cos t' + r \cos t . \sin t' = r' \sin p \cos p' + r' \cos p \sin p'$

Since r' and t' are to be supposed indefinitely small, we may substitute $1 = \cos t'$ and $t' = \sin t'$, &c Therefore $r \sin t + r't \cos t = r' . \sin p + r' p' \cos p$

(3) Hence by equat (1) $r t' . \cos t = r' p' \cos p$.

But as was shewn above, however small t' and p' are

$$t'^2 . p'^2 \quad r'^3 \quad r^3$$

By squaring equat (3), and substituting from this proportion

$$r' \cos^2 t = r . \cos^2 p,$$

or $r'(1-\sin^2 t) = r(1-\sin^2 p)$ Hence, by equat (1),

$$r'(1-\sin^2 t) = r\left(1-\frac{r^2}{r'^2}\sin^2 t\right)$$

Whence is easily deduced

$$\sin^2 t = \frac{r'^3 - r r'^2}{r'^3 - r^3} = \frac{r'^2}{r'^2 + rr' + r^2},$$

and therefore t is known.

This solution,[*] it is evident, answers both for a superior and an inferior planet

Problems in which Approximations are used

In many of the more useful investigations in astronomy, it is sufficient to make use of approximate solutions, as, for instance, in those for finding the effects of the precession of the equinoxes in right ascension and declination, for the effects of the aberration

[*] For a different solution of this Prop see Luby's Trig chap 4, part II.

of light in right ascension and declination, for the effects of parallax in latitude and longitude, and for a great variety of other problems, of which instances are to be found in every part of astronomy. These solutions, although only approximate, admit of all the accuracy that is necessary, and in general are obtained with much greater facility than exact solutions could be. In these solutions it often happens that we substitute the sine instead of the arc, radius instead of the cosine, the tangent instead of the arc, and conversely. It is therefore of some importance to know the limits of the differences of these quantities. Let a = an arc, s = its sine, c = its cosine, and t = its tangent, radius being unity; then

$$s = a - \frac{a^3}{6} + \&c.^{a}$$

$$c = 1 - \frac{a^2}{2} + \&c.$$

$$t = a + \frac{a^3}{3} + \&c.$$

Hence a small arc exceeds its sine by $\frac{a^3}{6}$ nearly,

the radius exceeds the cosine by $\frac{a^2}{2}$ nearly,

and the tangent exceeds its arc by $\frac{a^3}{3}$ nearly.

From whence by the trigonometrical tables it will easily be found that the difference between the arch of 1° 46′ and its sine is only 1″, and therefore in many cases one may be safely substituted for the other. The difference between the tangent 53′ and the arc is only 1″.

It is often of use in these problems to reduce a small arc, sine,

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[a] These expressions are easily proved by the diff. calc., and some writers have proved them by principles purely trigonometrical. See Luby's Trig. chap. 1 part II.

&c expressed in decimals of the radius (unity) to seconds, and the contrary. Which may be done as follows, let α = arc in decimals of the radius, a = the seconds in the arc, then as the sine of 1″ and the arc of 1″ are nearly equal, we have

$$\sin 1'' : 1'' :: a : \frac{a}{\sin 1''} = \alpha \quad \text{Hence also } a = \alpha \sin 1''$$

Prop XII *To find how much the time of rising of the sun or a star is advanced by refraction.*

Let RS (fig 57) represent part of the sun's parallel of declination, R the true place of the sun, when it appears rising at D. Then the time of rising is advanced by the angle SPR, the measure of which is the arc of the equator HL, and also the arc DR = the horizontal refraction

The small triangle SRD may be considered as a plane triangle and

$$RD : SR :: \sin RSD = \cos PSQ : \text{rad}$$

$$SR : HL :: \text{rad parallel} : \text{rad equat} :: \sin SP : \text{rad}$$

therefore $RD : HL :: \cos PSQ \times \sin SP : \text{rad}^2$,

$$\text{or } HL = \frac{RD}{\cos PSQ \times \sin SP \ (\cos \text{decl})}$$

But $\sin PSQ = \frac{\sin PQ}{\sin PS} = \frac{\sin \text{lat}}{\cos \text{decl}}$, and therefore the cos PSQ is known, and consequently HL, which, divided by 15, gives the time required

A somewhat more simple solution may be deduced from the above

$$\cos^2 PSQ = 1 - \frac{\sin^2 PQ}{\sin^2 PS} = \frac{\sin^2 PS - \sin^2 PQ}{\sin^2 PS} =$$

$$\frac{\sin (PS + PQ) \times \sin (PS - PQ)}{\sin^2 PS},\text{[a]}$$

[a] Trig p 23, formula 31

$$\text{therefore HL} = \frac{\text{RD}\,(1980'')}{\sqrt{(\sin(\text{PS}+\text{PQ}) \times \sin(\text{PS}-\text{PQ}))}},$$

$$\text{and } \frac{\text{HL}}{15}\,(\text{time}) = \frac{132''}{\sqrt{(\cos(\text{lat}-\text{decl}) \times \cos(\text{lat}+\text{decl}))}}$$

See remarks on this problem in Lalande's Astr. 3d edition, vol 3, art 4028. Cagnoli Trig. p 368.

Cor. 1. If RD be taken equal to the diameter of the sun, the time the sun takes in rising will be had.

Cor 2. If RD be taken equal to the difference between the horizontal parallax of the moon and the horizontal refraction, the time will be had of the retardation of the moon in rising.

Prop XIII. *Given the error in altitude, or in zenith distance, to find the error in time.*

Let rZ (fig 58) be the observed zenith distance, and Zs the true zenith distance, $rP = sP$ the polar distance. Join rs, and draw rn a portion of a parallel to the horizon. Then sn = error in zenith distance, and rPs = the error in the hour angle.

$$sn : sr :: \sin srn = \sin ZrP : \text{rad}$$

$$sr : rPs :: \sin rP : \text{rad}$$

hence $sn : rPs :: \sin rZP \times \sin rP : \text{rad}^2$

But $\sin ZrP \times \sin rP = \sin rZP \times \sin ZP$

$$\text{hence the error in time} = \frac{sn}{15 \sin rZP \times \sin ZP}$$

$$= \frac{\text{error in alt}}{15 \sin \text{azim} \times \cos \text{lat}}$$

Otherwise thus.—By spher. trig art. (106), p (67)

$$\cos rZ = \sin ZP \cdot \sin rP \cdot \cos ZPr + \cos ZP \cdot \cos rP$$

Hence

$$\sin rZ \,.\, d\,rZ = \sin ZP \cdot \sin rP \,.\, \sin ZPr \cdot d\,ZPr$$

$$= \sin ZP \cdot \sin rZ \,.\, \sin rZP \,.\, d \,.\, ZPr\,.$$

Consequently

$$d.\,ZPr = \frac{d.\,rZ}{\sin ZP \cdot \sin rZP},$$

or, as above,[a]

$$\text{error in time} = \frac{\text{error in alt}}{15 \sin \text{az} \cdot \cos \text{lat}},$$

Cor. In a given lat. the error in time is least when the sine of the azimuth is greatest, that is, when the azimuth is 90°, or when the body is in the prime vertical. Hence for finding the apparent time (art. 301) the observation should be made on or near the prime vertical.

Prop. XIV. *To find when the part of the equation of time, which arises from the obliquity of the ecliptic to the equator, is a maximum.*

If the sun moved equably in the ecliptic, the difference between its longitude (AL) (fig. 59) and the right ascension (AR) would be the equation of time. The longitude and right ascension are equal at the equinox and also at the solstice, and somewhere between, the difference is a maximum. It is evident the maximum will be when the difference ceases to increase, that is, when the increase of AL = the increase of AR.

Draw the circle of declination lr very near LR and meeting LR produced in the pole P.

Draw also lv a parallel of the equator. Then

$$Ll : lv :: \text{rad} : \sin ALR,$$

$$lv : Rr :: \sin Pv : \text{rad}. \quad \text{The sine } Pv = \cos LR \text{ nearly.}$$

Hence

$$Ll : Rr :: \cos LR \cdot \sin ALR = \frac{\cos A \times \text{rad}}{\cos LR}.$$

$$\text{Therefore } Ll : Rr :: \cos^2 LR : \cos A \times \text{rad}.^{a}$$

Hence when the equation is a maximum,

$$\cos^2 LR = \cos A \times \text{rad}$$

$$\text{or } \cos^2 \text{decl} = \cos \text{ob. ecl.}$$

[a] See note, page 180.

Otherwise thus —The difference of AR and AL is a *max* therefore, d AL — d. AR = 0. Now tan AR = cos A tan AL, hence

$$\frac{d\ \mathrm{AR}}{\cos^2 \mathrm{AR}} = \frac{\cos \mathrm{A}\ d\ \mathrm{AL}}{\cos^2 \mathrm{AL}},$$

Hence, since d AL = d AR, we have

$$\cos^2 \mathrm{AL} = \cos \mathrm{A} . \cos^2 \mathrm{AR}.$$

But

$$\cos^2 \mathrm{LR}\ \cos^2 \mathrm{AR} = \cos^2 \mathrm{AL}$$

Hence

$$\cos^2 \mathrm{LR} = \cos \mathrm{A}, \text{ as above.}$$

Prop. XV. *To deduce the sun's change in declination near the solstice*

Let O = ob ecl.

S = sun's distance from the solstice in seconds,

x = change in declination

By circular parts

sin ob ecl × sin long = sin decl,

or, $\sin \mathrm{O} \times \cos \mathrm{S} = \sin(\mathrm{O} - x) = \sin \mathrm{O} . \cos x - \cos \mathrm{O}\ \sin x$, (261) $\cos \mathrm{S} = 1 - \frac{1}{2}\mathrm{S}^2 \sin^2 1''$, $\sin x = x . \sin 1''$, and because x is very small compared with S, $\cos x = 1$

Hence by substitutions

$$\tfrac{1}{2}\mathrm{S}^2 \sin^2 1''\ \sin \mathrm{O} = x \sin 1''\ \cos \mathrm{O},$$

or $x = \frac{1}{2}\mathrm{S}^2 \sin 1''\ \tan \mathrm{O} = ,00000\ 1052\ \mathrm{S}^2$.

If D = sun's distance from solstice in degrees, $x'' = 13,63\ \mathrm{D}^2$, x will thus be had sufficiently exact for several days before and after the solstice, (see art 133)

Prop XVI. *To deduce the change of altitude of the sun or a star, when near the meridian, in a given time*

Let p = SP the polar distance of the object (fig 60), c = ZP the co-latitude, and $m + x$ = ZS the zenith distance, m being the zenith distance when on the meridian, and x = the change required

(1)..By trig. $\cos(m+x)=\cos p \cos c+\sin p \sin c \cos P$,

(2). $\cos(m+x)=\cos m-\sin x \sin m$, because x being supposed very small $\cos x=1$ nearly

(3) .$\cos m=\cos(p-c)=\cos p.\cos c+\sin p \sin c$

Hence equating the second members of (1), (2), and substituting for $\cos m$ as in (3),

there results $\sin x=\dfrac{\sin p \sin c(1-\cos P)}{\sin(p-c)}$.

But $1-\cos P=2.\sin^2\frac{1}{2}P$,
and $\sin x=x\sin 1''$ (page 261.)

$$\text{Hence } x \text{ (in seconds)} =\frac{2\sin p \sin c \sin^2\frac{1}{2}P}{\sin(p-c) \sin 1''}$$

$$=\frac{2\cos D \cos L \sin^2\frac{1}{2}P}{\sin m \sin 1''}$$

Where D and L = the declination and latitude respectively, and P = the time from the meridian reduced into space.

When L and D are nearly equal, and of the same kind, this expression can only be used at a very small distance of time from the meridian.

This problem is of much importance, when meridional altitudes are taken by the repeating circle

Prop XVII *To investigate nearly the quantity and law of atmospherical refraction.*

Let LI (fig 61) be a ray of light falling on the atmosphere at I, and refracted in the curvilineal course IS The object appears to a spectator at S in the direction ST, a tangent to the curve, and VST is the apparent zenith distance

The space in the figure between the concentric circles represents all the atmosphere, which has any effect on the ray of light, so that the light may be considered as passing out of a vacuum into this space.

If the surface of the earth were a plane, the different strata of air might be considered as parallel thereto, and by the principles

of optics, the refraction would be the same as would take place were the ray of light to pass from a vacuum into air of the same density as that at the surface. It is therefore evident that if we take into account the spherical form of the earth and atmosphere, the error resulting from the supposition of an uniform atmosphere will, necessarily, be very small compared with the change occasioned by considering the atmosphere spherical, provided that change be small.

Let $m : 1 ::$ sin of incidence : sin of refraction, when a ray of light passes from a vacuum into air of the density of that at the surface of the earth. Suppose all the air contracted into an uniform atmosphere, then SI is a right line. Let $HIL = i$, $SIC = r$, $VSI = z$, $SC = a$, the height of the uniform atmosphere $= l$, or $CI = a + l$.

$$a + l . a :: \sin z : \sin r$$

$$1 : m :: \sin r : \sin i$$

$$\text{Hence } \sin i = \frac{m\ a\ \sin z}{a + l} = m . \sin z \left(1 - \frac{l}{a}\right) \text{ nearly,}$$

$$\sin r = \frac{a\ \sin z}{a + l} = \sin z . \left(1 - \frac{l}{a}\right) \text{ nearly}$$

Let $i = r + R$, then R is the quantity of refraction, $\sin(r + R) = \sin i$

or because R is small, $\sin r + \cos r . \sin R = \sin i$,

or $\sin r + R\ \sin 1''\ \cos r = \sin i$,

substituting in this equation for $\sin r$ and $\sin i$ as above, also for

$$\cos r, \sqrt{\left(1 - \sin^2 z \left(1 - \frac{l}{a}\right)^2\right)} = \sqrt{\left(\cos^2 z + \frac{2\,l}{a} . \sin^2 z\right)}$$

$= \cos z \left(1 + \frac{l}{a}\ \tan^2 z\right)$ nearly, we obtain

$$R = \frac{\sin i - \sin r}{\sin 1'' . \cos r} = \frac{(m - 1)\ \sin z\ \left(1 - \frac{l}{a}\right)}{\sin 1'' . \cos z . \left(1 + \frac{l}{a} . \tan^2 z\right)}$$

$$=\frac{m-1}{\sin 1''}\ \frac{\tan z-\frac{l}{a}\ \tan z}{1+\frac{l}{a}\ \tan^2 z}$$

Hence by actually dividing

$$R=\frac{m-1}{\sin 1''}\left\{\tan z-\frac{l}{a}\ (\tan z+\tan^3 z)\ \&c\right\}$$

$$=\frac{m-1}{\sin 1''}.\tan z-\frac{m-1}{\sin 1''}\ \frac{l}{a}\ \tan z\,.\sec^2 z \text{ nearly.}$$

Taking $z=80^\circ$, $l=5$, and $a=4000$ miles, the second term (arising from the spherical figure of the atmosphere) $=10''$ nearly. If a were infinite, that is if the surface of the earth were a plane, this second term would vanish. Hence we may safely conclude, that as far as 80° zenith distance, the error arising from supposing the atmosphere of uniform density must be much less than 10″, and that consequently the above expression gives the refraction as far as 80° from the zenith with sufficient accuracy If we neglect the second term, the refraction will vary as the tangent of the zenith distance.

The exact experiments of MM. Biot and Arago have determined the value of $m-1=,0002946$ when the barometer is at 29,93 m (Metre) and Far. Therm, at 32°. From their experiments and the law of expansion of air it may be inferred that

$$\frac{m-1}{\sin 1''}=\frac{1,0375}{1+,002083\,(t-32)}\times\frac{b}{29,60}\times 57'',82 \text{ nearly, where } b \text{ is}$$

height of the barometer, and t that of Farenheit's thermometer.

When $t=50^\circ$ and $b=29,60$ inches, this expression gives $\frac{m-1}{\sin 1''}=57'',82$, a result independent on astronomical observations.

The French tables of refraction, by Delambre, founded on astronomical observations, give

$$\frac{m-1}{\sin 1''}=57'',72$$

By upwards of 500 observations made by myself with the eight feet astronomical circle,

$$\frac{m-1}{\sin 1''} = 57'', 56$$

The above value of R is quite exact enough for all observations as far as 80° from the zenith

From about 80° to the horizon the changes of refractions are so uncertain that observations are useless for the nicer purposes of astronomy.

The following tables will be found very convenient for computing the quantity of refraction for all the zenith distances not greater than 80° For the particulars of the construction of these tables, and for several investigations relative to astronomical refractions, references may be had to the 12th volume of the Transactions of the Royal Irish Academy; in which I have deduced the above expressions for refraction, independent of any hypothesis relative to the variations of density in the atmosphere.

By help of Table I. the first term of R is obtained The second table gives the second term of R, which near 80° has been slightly modified

TABLES FOR REFRACTION

Table I

Far Therm °	Logarithms	Far Therm °	Logarithms	Far Therm °	Logarithms
10	0 3283	34	0 3048	58	0 2827
11	0 3273	35	0 3039	59	0 2818
12	0 3263	36	0 3030	60	0 2809
13	0 3253	37	0 3020	61	0 2800
14	0 3213	38	0 3011	62	0 2791
15	0 3233	39	0 3001	63	0 2782
16	0.3223	40	0 2992	64	0 2773
17	0 3213	41	0 2983	65	0 2764
18	0.3203	42	0.2974	66	0 2755
19	0 3193	43	0 2965	67	0.2746
20	0 3183	44	0 2956	68	0.2737
21	0.3173	45	0 2946	69	0 2728
22	0 3163	46	0 2937	70	0 2720
23	0 3154	47	0 2928	71	0 2711
24	0 3144	48	0 2919	72	0 2703
25	0 3134	49	0 2910	73	0 2691
26	0 3124	50	0 2900	74	0 2685
27	0.3114	51	0 2891	75	0 2677
28	0 3105	52	0 2881	76	0 2668
29	0.3095	53	0 2872	77	0 2660
30	0 3086	54	0 2863	78	0 2652
31	0 3076	55	0 2854	79	0 2644
32	0 3067	56	0 2845	80	0 2636
33	0.3058	57	0 2836	81	0 2627

Logarithm in Table I. + log barom + log tan zenith distance = log approximate refraction

TABLE II.—BAROMETER.

Z D °	28,50 ″	29,00 ″	29,50 ″	30,00 ″	30,50 ″
80	10,5	10,7	10,9	11,1	11,4
79	8,1	8,3	8,5	8,5	8,9
78	6,3	6,4	6,6	6,7	6,9
77	5,1	5,2	5,3	3,5	5,6
76	1,1	4,2	1,3	4,1	1,5
75	3,4	3,1	3,5	3,6	3,7
74	3,0	3,0	3,1	3,1	3,2
73	2,5	2,5	2,6	2,6	2,6
72	2,1	2,1	2,2	2,2	2,2
71	1,8	1,8	1,9	1,9	1,9
70	1,5	1,5	1,5	1,6	1,6
69	1,3	1,3	1,3	1,4	1,4
68	1,2	1,2	1,2	1,2	1,2
67	1,0				1,0
66	0,9				0,9
65	0,8				0,8
64	0,7				0,7
63	0,6				0,6
62	0,6				0,6
61	0,5				0,5
60	0,5				0,5
58	0,4				0,4
56	0,3				0,3
54	0,3				0,3
52	0,2				0,2
50	0,2				0,2
45	0,2				0,2
40	0,1				0,1
30	0,0				0,0
0	0,0				0,0

Appr ref—Number Table II = refraction.

Example. Zenith dis 71° 26′, barom 29,76 inches, and ther 43°.

Log Tab I.....	0 2965	Appr ref	175″,4
Log barom.....	1 4736	Tab II.	2, 0
Log tan 71° 26	10 4738		
Log appr ref 175″,1	2 2439	Ref	173,4 = 2′ 53″,4

Prop. XVIII. *From given small variations in the longitude of a celestial object, and in the obliquity of the ecliptic, to deduce the variations of the right ascension and north polar distance*

1. For the effects of the variation in longitude. Let P (fig. 62, 1) be the pole of the ecliptic, N that of the equator, and F the fixed star or other object. Let FPM = the increase of longitude, the distance PM from the pole of the ecliptic remaining the same. Then FNM is the increase in right ascension, and drawing MR parallel to the equator, FR is the decrease of north polar distance.

$$FNM : RM :: \text{rad} : \sin MN$$
$$RM : FM :: \sin RFM = \cos PMN : \text{rad}$$
$$FM : FPM :: \sin PM : \text{rad}$$

Hence $FNM = \dfrac{FPM \cdot \sin PM \cdot \cos PMN}{\sin MN}$, (because $\sin PM \cdot \sin PMN = \sin PNM \cdot \sin PN$)

$$= \frac{FPM \cdot \sin PNM \cdot \sin PN \cdot \cot PMN}{\sin MN}$$

By spherical trigonometry[a]

$$\sin PNM \cdot \cot PMN = \cot PN \cdot \sin NM - \cos PNM \cdot \cos NM$$

Hence by substitution

$$FNM = FPM \cdot (\cos PN - \sin PN \cdot \cos PNM \cdot \cot NM) \qquad (a)$$

Also $FR = FM \cdot \sin FMR = FM \cdot \sin PMN$

$$= FPM \cdot \sin PM \cdot \sin PMN = FPM \cdot \sin PN \cdot \sin PNM. \qquad (b)$$

2. For the effects of the variation in the obliquity of the ecliptic. Let AP, PS (fig. 62, 2) represent the right ascension and declination of the point S, and AQ, QS the right ascension and declination when the obliquity of the ecliptic is changed by the angle PAQ. Then when this angle is very small, PR = the change of north polar distance nearly, and RQ = the change of right ascension nearly.

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[a] Trig. p. 69, art. 112.

$$\sin RP \quad \sin A \qquad \sin AP \quad \sin ARP,$$

or RP = A . sin rt. ascension nearly (c)

Also $\cot AR \tan RP = \cos ARP = \tan RQ \cdot \cot SR$, hence $RQ = RP \cdot \cot AR \cdot \tan SR$,

and RQ = A . cos (rt. asc.) cot (north polar distance) (d)

Application of the preceding Prop.

The above proposition enables us to deduce the *apparent* right ascension and north polar distance from the mean, as affected by precession, lunar nutation and solar nutation.

The actions of the sun and moon causing[a] a change of place of the intersections of the ecliptic and equator, the longitude of each object is changed by the same quantity. The same action also occasions the obliquity of the ecliptic to be variable. The action of the planets also changes the plane of the earth's orbit, and therefore the intersections and inclination of the ecliptic and equator.

Let L = the mean longitude of a star in the beginning of 1820. Then for the time 1820 + t, t being taken negatively or positively,

App. long. = L + 50″,19 . t − 17″,30 sin long. ☽'s node + 0″,21 . cos 2 long. ☽'s node − 1″,25 . sin 2 long ☉ − 0″,21 . sin 2 long ☽ . (e)

App. obliq. of ecl. (O), for 1820 + t = 23° 27′ 47″ − 0″,45 t + 9″ 25 cos long. ☽'s node − 0″,09 . sin 2 long. ☽'s node + 0″,54 . cos 2 long ☉ + 0″,09 . cos 2 long ☽ . .. .. (f)

The form of these quantities has been obtained by investigations in physical astronomy, the larger coefficients and 0″,45 have been obtained by observation.

In the above the term 50″, 19t increased by 0″, 13t serves for determining what is usually called the effect of *precession in right ascension and north polar distance*. When so increased it is the mean effect (luni-solar) of the sun and moon. The part 0″,

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[a] Art. 90, &c.

13t is occasioned by the action of the planets, and diminishes the effect of the actions of the sun and moon. This affecting only the ecliptic, does not affect the north polar distance.

The terms depending on the longitude of the moon's node serve for determining the effect of what has been called the *lunar nutation* The terms depending on the longitude of the sun serve for determining what has been called the *solar nutation* The terms depending on twice the longitude of the moon, and twice the longitude of the sun, are too small to be noticed, except in the nicest researches.

I *Precession in right ascension* (A), *and north polar distance* (NPD).

Taking the angle FNM (the change of longitude in the preceding problem $= 50'', 32\,t$, t not exceeding a few years, by equation (a).

The luni-solar precession in right ascension $=$

$$50'', 32\,t\ (\cos O + \sin O\ \sin A\ \cot NPD) =$$
$$46'', 18\,t + 20'', 04\,t \,.\, \sin A \,.\, \cot NPD,$$

from which subtracting $0'', 13\,t$, the general precession in right ascension will be had

By equation (b) the precession in NPD $=$

$$-50'', 32\,t \,.\, \sin O \,.\, \cos A = -20'', 04\,t \,.\, \cos A.$$

Cor 1. The precession in right ascension will be subtractive, when $\sin A\ \cot NPD$ is negative and greater than $\cot O$, which can never happen but when $\cot NPD$ is greater than $\cot O$ Therefore the right ascension of every star, the polar distance of which is greater than 23° 28′, is always increased by precession.

Cor. 2. Precession affects equally the north polar distance of every star having the same right ascension

II. *Lunar nutation in right ascension and north polar distance*

Let the angle $FNM = -17'', 30 \,.\, \sin N$, N being the longitude

of the moon's node, and the change of obliquity $= 9, 25 \cos N$ See equations (*e*) and (*f*) Then by equations (*a*), (*d*)

Nutation in right ascension $=$

$$-17'', 30 \sin N \ (\cos O + \sin O \ \sin A \ \cot NPD),$$
$$+9'', 25 \ \cos N \ \cos A \ \cot NPD =$$
$$-15, 87 \ \sin N - [8'' \ 07 \ \cos (A - N) + 1'', 18 . \cos (A + N)] \ \cot NPD$$

By equation (*b*) and (*c*),

The nutation in north polar distance $=$

$$17'', 30 \ \sin N \ \sin O \ \cos A - 9'', 25 \ \cos N \ \sin A,$$
$$-8'', 07 \ \sin (A - N) - 1'', 32 . \sin (A + N).$$

III *Solar nutation in right ascension and north polar distance*

Let the angle $FNM = 1'', 25 \ \sin 2 S$, S being the longitude of the sun, and the change of obliquity $= 0, 54 . \cos 2 S$, and we obtain by a process similar to that in II

Solar nutation in right ascension $=$

$$-1'', 15 \ \sin 2S - [0, 51 . \cos (A - 2 S) + 0, 02 \ \cos (A + 2 S)] \ \cot NPD$$

Solar nutation in north polar distance $=$

$$-0'', 51 \ \sin (A - 2 S) - 0'', 02 \ \sin (A + 2 S)$$

The solar nutation has also been called the semi-annual equation, because depending on twice the sun's longitude it goes through its period in half a year.

Prop. XIX. *To deduce the effect of the aberration of light on the right ascension and declination of a star*

Let S (fig 63) be the star, ED the equator, N its pole, MELH the ecliptic, and MS a great circle perpendicular to the circle of declination NSHD

Let L be the point of the ecliptic towards which the earth is moving, which is always 90° behind the place of the sun Take

Sp so that $\sin Sp : \sin SL :: $ vel. of earth : vel. of light :: $\sin 20''\frac{1}{4}$: rad, or which comes to the same,

$$Sp : 20''\tfrac{1}{4} :: \sin SL : \text{rad},$$

then p is the apparent place of the star as affected by aberration (art. 281 and 283).

Draw pq parallel to the equator, and then Sq = aberration in declination. Also pNq = aberration in right ascension. Let $n = 20''\frac{1}{4}$.

$1.\ pq : Sp :: \sin pSq : \text{rad}$

$Sp : n :: \sin SL : \text{rad}$

$pNq : pq :: \text{rad} : \sin Nq$, or $\sin NS$ sufficiently near.

Hence $pNq = \dfrac{n \cdot \sin pSq \cdot \sin SL}{\sin NS} = \dfrac{n \cdot \sin H \cdot \sin LH}{\sin NS}$, or aberration in R. ascen. $= \dfrac{20''\frac{1}{4} \cdot \sin H \cdot \sin LH}{\cos \text{decl.}}$

$2.$ Aberration in decl $= Sq = Sp \cdot \cos pSq = n \cdot \sin SL \cdot \sin MSL$ $= 20''\frac{1}{4} \cdot \sin M \cdot \sin ML$.

From the above expressions very convenient practical formulas may be deduced.

As $\dfrac{n \cdot \sin H}{\cos \text{decl.}}$, and $n \cdot \sin M$ are constant for a given star, the aberration in right ascension varies as the sine of LH, and the aberration in declination varies as the sine of LM.

1. Let L = the sun's longitude, then, because the aberration in right ascension (A) varies as $\sin LH =$

$$\sin(EH - (L - 90^\circ)) = \sin(90^\circ - (L \backsim EH)) =$$

$\cos(L \backsim EH)$, we may express it by $m \cdot \cos(L \backsim K)$, and supposing m positive, $K = EH$ will be the sun's longitude when the aberration is a maximum and positive. K and m may be found in the following manner:

$$\tan K\,(EH) = \frac{\tan ED}{\cos HED} = \frac{\tan A}{\cos O}.$$

When $L = 90^\circ$, the point L is in E.

Then $m(\cos 90-K)=m\,.\sin K=$ ab in R ascens $=$

$$\frac{-20''\frac{1}{4}\ \sin H\,.\sin EH}{\cos D}=\frac{-20''\frac{1}{4}\ \sin A}{\cos D}$$

Hence $m=\dfrac{-20''\frac{1}{4}\ \sin A}{\cos D\ \sin K}$

It only remains to be known to what quadrant K belongs: tan K has the same sign as tan A

Because m is positive $\dfrac{\sin A}{\sin K}$ must be negative, cos D being always positive Therefore A and K must be in opposite semicircles, and, their tangents having the same signs, they must be in opposite quadrants.

2. Again, because the aberration in declination varies as sin ML $=\sin(ME+L-90)=\cos(180-ME-L)$, we may express it by $m'\cos(L\backsim K')$, and supposing m' positive, $K'=180^\circ-ME$ will be the sun's longitude when the aberration is a maximum and positive

K′ and m' are found in the following manner:

$$\tan K'=-\tan ME.$$

By spherical triangle[a] PEM

$$\tan ME=\frac{\sin PE}{\cot P\ \sin E+\cos PE\ \cos E}$$

Therefore, because $\cot P=-\cot SPD$ $(=\text{decl})$

$$\tan K'=\frac{\cos A}{\cot D\,.\sin O-\sin A\ \cos O}$$

Also $\sin M\,.\sin ME=\sin PE\,.\sin P=\cos A\ \sin D$

Therefore $m'=\dfrac{-20''\frac{1}{4}\ \cos A\ \sin D}{\sin K'}$ The quadrant to which K′ belongs is thus determined, the sign of tan K′ is known from its value above given. The sign of m' being positive, $\cos A\,.\sin D$ and $\sin K'$ must have different signs, but knowing the signs of tangent and sine of an arc the quadrant is known

[a] Cagnoli Trig p 270. Luby's Trig p 73 art. 120

The quantities m, m', K, K' being constant for the same star, render this a very concise method of computation, when the aberrations of the same star for several days are required, or when tables of the aberrations of a given star are required.

Indeed m and m' are not strictly constant on account of the variable velocity of the earth, but the variation is so small that usually it is not considered.

When a single place of a star is required, then general tables are more convenient [a]

The three last problems containing the effects of refraction, precession, nutation solar and lunar, and aberration of light, are of constant use in practical astronomy

Prop XX *Given the mean anomaly of a planet, to find the true anomaly*

Let the semiaxis major of the planet's orbit $=1$, its eccentricity $=e$, $m=$ the mean anomaly, $z=$ the eccentric anomaly, and $y=$ the true anomaly

Find the arc d, so that

$$(1)\,..\tan d=\frac{1-e}{1+e}\tan\tfrac{1}{2}\ m,$$

then $\frac{1}{2}m+d=$ eccentric anomaly nearly, (art. 233) for which substitute p, and let $p+v=z$, the eccentric anomaly

Now by the same article

$$(\text{fig } 33)\ \text{ACL}\,(m)=\text{ACI}\,(p+x)+\text{LCI}$$

Now LC arc LI $=2$ area LCI $=2$ area SIC $=$ SC CI . sin ICD,

or

$$\text{Arc LI}=\text{SC}\ \sin \text{ICD},$$

Hence the seconds in LI, or the angle $\text{LCI}=\dfrac{\text{SC}\,.\sin\text{ICD}}{\sin 1''}$

$$\text{Therefore } m=p+x+\frac{e\sin(p+x)}{\sin 1''}=$$

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[a] Woodhouse's Astr. p. 466, &c Conn. des Temps 1810, p. 422 and 442

$$p + x + \frac{e \sin p}{\sin 1''} + e x \cos p, \text{ because } x \text{ is small.}$$

If, to adapt this expression to logarithms, we use the auxiliary arc b

$$\text{so that } e \cos p = \cos b,$$

$$\text{then } 1 + e \cos p = 1 + \cos b = 2 \cos^2 \tfrac{1}{2} b$$

$$\text{Also let } p + \frac{e \sin p}{\sin 1''} = m'$$

$$\text{Then } m = m' + 2x \cos^2 \tfrac{1}{2} b$$

$$(2) \ldots \text{or } x = \frac{m - m'}{2 \cos^2 \frac{1}{2} b}.$$

The eccentric anomaly $(p + x)$ thus found will be sufficiently exact to give the true anomaly to less than a second for all the planets And by repeating the process, using this last eccentric anomaly instead of p, the eccentric anomaly will be obtained in the most eccentric orbits. To find the true anomaly, the following equation *has long been used

$$(3) \ldots \tan^2 \tfrac{1}{2} y = \frac{1 - e}{1 + e} . \tan^2 \tfrac{1}{2} z.$$

The solution furnished by the equations (1), (2), (3), appears to be better adapted to practice, and it affords a more exact result than the solutions usually given of this problem The particulars of this method of solving this important problem, and the practical rule resulting, are stated in the Transactions of the Royal Irish Academy, vol. ix p. 143, &c.

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* This may be proved as follows —Referring to fig 33, we have $e + \cos z = SD = PS \cos y = \frac{1 - e^2}{1 - e . \cos y} \cos y$ See Lloyd's Analytic Geom p 110 art 51

Hence $\cos z = \frac{\cos y - e}{1 - e \cos y}$. Then subtracting each side of this equat from and adding it to unity, and by division there results $\frac{1 - \cos z}{1 + \cos z} = \frac{1 + e}{1 - e} \times \frac{1 - \cos y}{1 + \cos y}$

Hence by Trig p 24, form. 52, we have $\tan^2 \frac{1}{2} y = \frac{1 - e}{1 + e} \tan^2 \frac{1}{2} z$

Prop. XXI. *Given the horizontal parallax of the moon and apparent altitude, to find the parallax in altitude*

2 *Given the hor parallax and true altitude of the moon, to find the parallax in altitude.*

3 *From the apparent altitude and horizontal parallax, to find the apparent diameter of the moon*

1 Let the apparent zenith distance $= z$, the true zenith dist $= v$, the parallel in alt $= p$, the horizontal parallax $= h$, $SC = 1$, $CL = d$ (fig 64).

then sin VSL (z) . sin SLC (p) . : d : 1

$$\text{or } \sin p = \frac{\sin z}{d},$$

when $z = 90°$, $p = h$,

$$\text{therefore then } \sin h = \frac{1}{d},$$

consequently $\sin p = \sin h . \sin z$,

or sufficiently near, $p = h \sin z$

The equatorial parallax, that is, the horizontal parallax at the equator, is given for noon and midnight at Greenwich in the Nautical Almanac, and therefore the horizontal parallax may be found for any latitude by diminishing the horizontal parallax in proportion of the distance from the centre of the earth to the equatorial semidiameter

2. As above, $\sin p = \sin h . \sin z = \sin h \sin (v + p) = \sin h . \sin v \cos p + \sin h \cos v \sin p$

$$\text{Therefore } \frac{\sin h \sin v}{1 - \sin h . \cos v} = \frac{\sin p}{\cos p} = \tan p$$

From which may be found by a particular process[a]

$$p = \frac{\sin h \sin v}{\sin 1''} + \frac{\sin^2 h \sin 2v}{\sin 2''} + \frac{\sin^3 h \sin 3v}{\sin 3''} + \&c.$$

3. By what appears the most accurate result, the moon's dia-

[a] Astron Delambre, tome 1. p 214 Trig part II ch. 4.

meter, as seen from the centre of the earth, horizontal equatorial parallax : .5455 10000 Let this ratio be expressed by the ratio of m 1

The apparent diameter of the moon from S : apparent diameter from centre : CL . SL . : sin z sin $(z-p)$ If the horizontal equat parallax be expressed by e, then the moon's apparent diameter as seen from the centre $= me$

$$\text{Hence the app. diam from S} = \frac{m . e \sin z}{\sin (z-p)}.$$

Prop XXII *To find the moon's parallax in longitude and latitude, having given the horizontal parallax.*

Let Z (fig. 65) be the zenith; P the pole of the ecliptic; T the true place of the moon, and A its apparent place in the same vertical circle ZTA Then TPA is the parallax in longitude, and AQ is the parallax in latitude, QT being parallel to the ecliptic.

PZ the distance of the pole of the ecliptic from the zenith, which is equal to the height of the nonagesimal degree, and also the longitude of the zenith point, are found by Prop 5

Hence ZPT, the difference between the true longitude of the moon and the longitude of the zenith point is known. Therefore in the triangle ZPT are known ZP, PT, the distance of the moon from the pole of the ecliptic, and the included angle ZPT. Hence TZ, the true zenith distance of the moon, and ZTP may be computed

TZ being found, TA the parallax in altitude will be found by the last problem

Then in the triangle ATP are known TP, AT, and the included angle ATP, hence the side AP and the angle APT may be found.

This solution of the problem has the advantage of being exact, and also of requiring the solution of two oblique angled triangles of exactly the same data

It has the disadvantage of requiring seven places of logarithms.

The parallaxes in longitude and latitude may be also found by approximate formulæ, in the computation of which five places of

logarithms are sufficient[a] But such formulæ are in some respects inconvenient, and perhaps on the whole have not much advantage over the above

The above is on the supposition that the earth is spherical An allowance is made for its spheroidical figure, as Mayer first showed, by simply diminishing the latitude of the place by the angle contained between a perpendicular to the surface and a line drawn from the place to the centre of the earth This angle is easily computed, and is, taking the excess of the diameters $\frac{1}{330}$, in the lat of Dublin $= 10'.\ 0''$ No other change is necessary than using the latitude thus diminished in computing the nonagesimal.

Prop XXIII *To find the longitude of a place by observing the difference of the times of the transits of the moon and a fixed star, the same observations having been made at a place, the longitude of which is known* (art. 310).

As the observations require a transit instrument, and as the clock used with the transit instrument generally shews sidereal time, the difference of times is supposed to be sidereal time

If the moon did not move, the difference of times of its transit, and of that of a fixed star would be the same at each place The difference of the differences arises entirely from, and is equal to the increase (I) of the moon's right ascension in time, in the interval between the passages of the moon over the meridian of each place

Hence if we know the increase (N) of the moon's right ascension in one hour of sidereal time,

$N : I :: 1^h : X =$ the angle in time described by the western meridian in the interval of the passages of the moon $=$ diff. of long. $+$ I.

$$\text{Hence diff. of long} = X - I = \frac{I}{N} - I.$$

By the Nautical Almanac the moon's right ascension is given at

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[a] Vince's Astr vol. I. p 307 Woodhouse's Astr. p 366 and 367. Astron. Delambre, tom. 2, p. 408.

apparent noon and midnight to minutes of a degree. From thence its increase in an hour of sidereal time may be found.

The increase for that hour should be used, the middle of which is nearly in the middle of the interval which includes the observations

This method is susceptible of great accuracy, and when the places differ much in longitude, the increase of right ascension of the moon should be computed more accurately than can be done from the right ascensions given in the Nautical Almanac But the right ascensions can be obtained from the latitudes and longitudes given in the same work as accurately as can be desired, by the converse of Prop. 4.

The best method then would be to assume the difference of longitude $= L'$, which can be done nearly, and then compute the increase (E) of the moon's right ascension in the sidereal time L' and then

$$E . I : : L' . X = \frac{IL'}{E},$$

and the exact diff of longitude

$$= \frac{IL'}{E} - I.$$

But this exactness is only necessary when the places differ considerably in longitude.

The moon's limb is observed by a transit instrument and not the centre, but this makes no difference except when the places differ much in longitude. It may then be necessary to make an allowance for the moon's alteration of distance, which changes its apparent diameter, and also for its change of declination, which changes its semi-diameter in right ascension.

A mean of many results obtained by observing the transits of the moon before and after opposition, when different limbs are enlightened, will give great accuracy.

Prop XXIV. *To find the longitude of a place from the comparison of the observations of the beginning or end of an*

eclipse of the sun, made at two places, the longitude of one of which is known.

1. At one of the places let, at the time of observation of the beginning of the eclipse, the sun's semi-diameter + the moon's semi-diam = S The moon's semi-diameter is to be increased according to its altitude (Props 21 and 22)

Let the true latitude and longitude of the moon at the time of observation be found from the Nautical Almanac. These should be corrected for the errors of the tables if known by observation.

The apparent latitude (L) of the moon and the parallax in longitude are to be found by Prop. 22 At the same time the apparent diameter of the moon may be found

By the right angled triangle formed by S, L and the apparent difference of longitude of the moon and sun, we can find this apparent diff its log being equal to the log

$$\sqrt{(S^2-L^2)} = \tfrac{1}{2}\log(S+L) + \tfrac{1}{2}\log(S-L).$$

The apparent difference of longitude between the sun and moon being known, the true difference is known, by help of the moon's parallax in longitude.

Hence the interval between the time of observation and the time of conjunction as seen from the centre of the earth, is known, by the help of the sun's and moon's motions in longitude given in the Nautical Almanac. Therefore the time of the true conjunction is known at one of the places

2. By a similar proceeding the time of the true conjunction will be known from the observation of the beginning at the other place.

The difference of these times is the difference of longitudes

The mode of proceeding is the same for determining the difference of longitudes by observations of the end of the eclipse [a] From

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[a] It is convenient to carry on the computation so that the effect of any errors in the *data* on the result may be known, and thus the degree of the accuracy of the conclusion estimated This may be done as follows:

Let ds = the error in the sum of the semi diameters,

the above also may be understood the mode of proceeding in determining the longitude by an occultation of a star

Prop. XXV. *Considering the earth as an oblate spheroid, the equatoreal diameter exceeding the polar diameter by only a small quantity, to find the arc of the meridian intercepted between the equator and a given place.*

APN (fig. 66) is the elliptic quadrant, N the pole, AOM the circular quadrant, and OT, TPR are tangents.

Let $AC=m$, $CN=n$, the lat of the place $P(=OPR)=l$, $TOB=c$, $AB=x$, and $AP=z$,

$$\cot l \,.\, \cot c :: PB : OB :: n : m,$$

$$\text{whence } \cot c = \frac{m}{n} \cdot \cot l = \frac{\cot l}{1-s}, \text{ making } s = 1 - \frac{n}{m},$$

$$\text{Hence } \frac{dc}{\sin^2 c} = \frac{1}{1-s} \cdot \frac{dl}{\sin^2 l}.$$

Also $x = m - m \,.\cos TOB = m - m \cos c$ Hence $dx = m \,.\, dc$ $\sin c$, but $dx = dz \,.\sin l$, therefore $dz = \frac{m}{1-s} \cdot \frac{dl \sin^3 c}{\sin^3 l}$

To develope the right hand side of this equation, regarding only the first power of the small quantity s, let $c = l = k$ Then sin

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$dl=$ the error in the latitude of the moon,

$dp=$ the error in the horizontal parallax

Then if the computation be carried through with these quantities annexed to the sum of the semi-diameters, latitude of the moon, and horizontal parallax; we shall have, calling T the time of conjunction computed as in the above solution from the eastern observation, and T' that from the western.

The time of conjunction at eastern place . $=T+a\,ds+b\,dl+c\,dp$,

The time of conjunction at western place. $=T'+a'ds+b'dl+c'dp$

Where a, b, c, a', b', c', are coefficients resulting from the computation

Hence the diff of longitude $=$

$T-T'+(a-a')\; ds+(b-b')\; dl+(c-c')\,.\,dp.$

From the magnitude of the coeff $(a-a')$, &c, the accuracy of the result may be estimated Thus if $(a-a')$, &c, be very small quantities considerably less than unity, the observations are adapted to give the difference of longitude to considerable accuracy Vid Conn. des Tem. 1811. p 458.

$$k = \sin OTP = \sin TPO\ \frac{OP}{OT}.$$

$$\text{But } \frac{OP}{OT} = \frac{OP}{OB}\ \frac{OB}{OT} = \frac{m-n}{m}.\cos c = s\ \cos(l-k)$$

Thus $\sin k$ is of the same order of magnitude as s, and therefore (since $\cos(l-k) = \cos l . \cos k + \sin l\ \sin k = \cos l + k . \sin l$), regarding only the first powers of s, we have $k = s . \sin l\ \cos l$.

Now $\sin c = \sin(l-k) = \sin(l - s . \sin l . \cos l) = \sin l - s . \sin l . \cos^2 l$. Hence we have

$$dz = \frac{m . dl}{1-s}. (1 - 3 s . \cos^2 l)$$
$$= m . dl . (1 - \tfrac{1}{2}s - \tfrac{3}{2} . s . \cos 2 l).$$

Hence integrating

$$z = m, (l'' . \sin 1'' - \tfrac{1}{2} s l . \sin 1'' - \tfrac{3}{4} . e . \sin 2 l),$$

l'' denoting the latitude in seconds

Prop. XXVI. *Given* (*a*) *the length of the arc of the meridian between* L′ *and* L′+D, *and* (*b*) *the length of the arc between* L″ *and* L″+D, *to find the equatoreal and polar diameters* (*m*) *and* (*n*)

We first find ms or $m-n$, and then m as follows; s is called the compression. Let D″ denote the seconds in D; then by the preceding prop.

$$a = m . \{D'' . \sin 1'' - \tfrac{1}{2}s . D'' . \sin 1'' - \tfrac{3}{4} . s . \sin(2L' + 2D) + \tfrac{3}{4} . s . \sin 2L'\} \qquad (1)$$
$$b = m . \{D'' . \sin 1'' - \tfrac{1}{2}s . D'' . \sin 1'' - \tfrac{3}{4}\ s . \sin(2L'' + 2D) + \tfrac{3}{4} . s\ \sin 2L''\}$$
$$b - a = \tfrac{3}{4}\ sm . \{\sin(2L' + 2D) - \sin 2L' - \sin(2L'' + 2D) + \sin 2L''\}.$$

$$\text{But}^{a} \sin(2L' + 2D) - \sin 2L' = 2 \sin D . \cos(2L' + D)$$
$$\sin(2L'' + 2D' - \sin 2L'' = 2 \sin D . \cos(2L'' + D)$$
$$\cos(2L + D) - \cos(2L'' + D) = 2 \sin(L' + L'' + D).\ \sin(L'' - L')$$

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[a] See Trig. p. 21, formulæ (13), (14)

Hence

$$b - a = 3m\ s . \sin D\ \sin(L'' - L')\ \sin(L'' + L' + D)$$

Whence

$$ms = \frac{b - a}{3\ \sin D . \sin(L'' - L') . \sin(L'' + L' + D)}$$

By equation (1)

$$m = \frac{a}{D'' . \sin 1''} + \tfrac{1}{2} m . s + \tfrac{3}{2} ms\ \cos(2L' + D) . \frac{\sin D}{D''\ \sin 1''}$$

If D be a small arc e g. 1°, $\frac{\sin D}{D''\ \sin 1''} = 1$ nearly,

and $m = \frac{a}{\sin D} + \frac{1}{2} ms + \frac{3}{2}\ ms . \cos(2L' + D)$. Also $n = m - ms$

Example Colonel Mudge found the degree commencing lat 51° 32′ north = 121640 yards. Major Lambton found the degree commencing lat. 12° 33′ north = 120975 yards

By the above formulæ, $m - n = 22167$ yards, $m = 6972238$ yards = 3961,5 miles, $n = 6950071$ yards = 3948,9 miles, and $s = \frac{1}{313}$